Research paper

Diameter and Height Distributions of Natural Even-Aged Pine Forests (*Pinus sylvestris*) in Western Khentey, Mongolia

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【Summary】

The purpose of this study was to find a suitable probability density function (PDF) to model the diameter at breast height (dbh) and height distributions of even-aged pine (*Pinus sylvestris* L.) forests. For the study, three different age-classes (AGs) of pine forests were used. Burr, Dagum, and Johnson SB distributions were applied due to their flexible properties. Result showed that dbh distributions of the 10~15- (AG1) and 40~45-yr (AG2) stands were left-tailed, while the 60~65-yr (AG3) stand was normally skewed. Height distributions of the AG1 and AG3 stands were left-tailed, while that of the AG2 stand showed no obvious distribution shape, due to its discrete height distribution. A distribution study revealed that in left-tailed forests, dbh and height distribution shapes were best approximated by the Dagum distribution. In the case of the normal distribution shape, the Johnson SB was better than the Burr and Dagum ones. Based on these results, we concluded that dbh distributions of even-aged AG1 and AG2 forests were heavily left-tailed, and the forest structure tended to normal for the AG3 forest. The height distribution is left tailed (AG1 and AG3) if a forest's height growth is not constrained by space, while it will become discrete in a high-density stand (AG2).

Key words: diameter at breast height, height, left-tailed distribution, Scots pine (*Pinus sylvestris* L).

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研究報告

蘇格蘭松同齡天然林的直徑與樹高分佈

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摘 要

本文以蒙古國Khentey省的西部天然同齡林為研究地區，利用Burr、Dagum以及Johnson SB機率密度函數模擬配適蘇格蘭松(*Pinus sylvestris* L.)不同齡級的林分胸高直徑以及樹高的分佈模式。結果顯示：10年生以及40年生齡級的林分直徑分佈為向左偏斜的分佈(negatively skewed distribution)，60年生的齡級則為常態分佈(normal distribution)；10年生以及60年生齡級的林分樹高分佈為向左偏斜的分佈，40年生齡級的林分的立木樹高呈離散的分佈，並無明顯的分佈特徵。依據配適函數模型的結果，蘇格蘭松同齡天然林的林分直徑以及樹高分佈可以利用Dagum、Burr、Johnson SB函數配適之，但由模型統計值顯示Dagum函數模型對於左偏分布的林分結構配適效果較為理想，而Johnson SB函數對於常態的林分結構配適效果較佳。林分結構資料顯示：若蘇格蘭松同齡天然林分有足夠的林分空間以供生長，林分的直徑結構會呈現很明顯的左偏分布，經過40年的生長期，會發展成為常態分佈；如果林分的生長空間有限，則林分的樹高結構將會由左偏現象發展為不規則結構。60年齡級以下的林分(AG1、AG3)樹高結構為左偏分布，但是如果林分密度太高將造成離散型結構，無明顯的函數分布型態。

關鍵詞：胸高直徑、樹高、左偏分佈、蘇格蘭松。

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INTRODUCTION

Information concerning the size-class distribution of a forest stand such as the diameter at breast height (dbh) and height classes is generally required for effective forest management planning. dbh and height distributions are particularly valuable for volume estimations of forest stands and yield planning. Even for unmanaged forest stands, dbh and height information can be used as standards for comparison of different types of managed stands. Thus, detailed modeling of the distributions of dbh and height classes for forest-vegetational types is required.

A wide range of probability density functions (PDFs) have been used in forestry to model tree dbh and height structural distributions (Bailey and Dell 1973, Hafley and Schreuder 1977, Gove et al. 2008, Wang et al. 2010, Tsogt and Lin 2013) and age distributions (Lin et al. 2007). Regular shaped (unimodal) distributions may take different shapes. A suitable PDF model that can express empirical distributions of diameter and height parameters of stands should be carefully derived using an appropriate theoretical distribution function. Some flexible PDF algorithms such as Beta, Burr, Dagum (inverse Burr), Gamma, Johnson SB, and Weibull have been widely used for this purpose in forestry.

Weibull is generally the most favored distribution in forestry as it is easy to use and able to describe both positive and negative skewness. A comparison study of lognormal and Weibull distributions on a regularly shaped birch forest dbh distribution structure showed that Weibull was superior (Tsogt et al. 2011c). However, that study included no other distribution functions. According to Lindsay et al. (1996), the Burr distribution outperforms the Weibull distribution in its ability to drive the dbh distribution, while Dagum has the ability to fit a rotated sigmoid dbh distribution (Gove et al. 2008). Both Weibull and Burr distributions are Dagum family distributions, and Dagum distributions have a wider region of applicability then either of the other two (Lindsay et al, 1996). The Burr (Zimmer and Burr 1963) and Dagum (1977) distributions are together called Burr-type distributions and were introduced to forest research by Lindsay et al. (1996). These distributions are inherently more flexible, because they can cover a much larger area of the skewness-kurtosis plane than the Weibull distribution (Rodriguez 1977, Tadikamalla 1980, Lindsay et al. 1996, Gove et al. 2008). That is because Burr-type distributions can approximate normal, gamma, lognormal, exponential, logistic, and several Pearson-type distributions (Rodriguez 1977, Tadikamalla 1980).

Hafley and Schreuder's (1977) first introduced the Johnson SB distribution in the forest literature, and their study showed that the Johnson SB distribution gave the best performance, while the normal, lognormal and gamma distributions were inferior to the Weibull and beta distributions in terms of their general performance over a variety of even-aged stands. Generally beta was the second best fitting distribution, and Weibull was the third best. From the viewpoint of practical applications, they believed that the Johnson SB distribution has important advantages over the beta distribution, in that it spans a slightly broader range of the skewness-kurtosis space than the beta distribution (i.e., it also covers the region between the lognormal and gamma). Since then, it has been widely used for modeling forest dbh and height distributions (Hafley and Buford 1985, Knoebel and Burkhart 1991, Zhou and McTague 1996, Kamziah et al. 1999, Li et al. 2002, Scolforo et al. 2003, Zhang et al. 2003). Later, its parameterization method was improved by Rennols and Wang (2005). The Johnson SB distribution covers a different region of the skewness-kurtosis plane from those of the Burr and Dagum distributions (Johnson 1949, Hafley and Schreuder 1977).

Understanding forest structures is helpful in determining forest productivity (Yang and Feng 1989, Chiu et al. 2010, Huang et al. 2012). Recently, some studies focused on the dbh and height distributions of species-specific forest stands such as birch (Tsogt et al. 2011c) and larch (Tsogt et al. 2011a, b, Tsogt and Lin 2013) in Mongolia. But, no one has studied dbh and height distribution of pine forests. The purpose of this study was to derive an appropriate distribution for modeling the dbh and height distributions of even-aged pine forests using the Burr, Dagum, and Johnson SB distributions.

MATERIALS AND METHODS

Study sites

The distribution models investigated in this paper were fitted to dbh and height measurements of 3 forest stands: P1 (49°45'86"N, 109°04'40"E at 920 m elevation) and P2 (49°46'16"N, 107°01'12" E at 888 m elevation) at Khuder soum Selenge aimag, and P3 (49°11'25"N, 106°39'02"E at 1145 m elevation) at Shariin gol soum Darkhan-Uul aimag. The species found at these study sites is a pure forest of Scots pine (*Pinus sylvestris* L.). Scots pine forests dominate the area of this study which belongs to the Western Khentey forest-vegetational province. It is also a common species of subtaiga forests of the Western Khentey by elevation-belt zonation (Tsedendash 1993).

The climate of the Khentey Mountains is characterized by the Asiatic anticyclone in winter, which is typically centered southwest of Lake Baikal and causes dry and cold winters with mean January temperatures of as low as -28 to -23℃ (Tsedendash 1993). Mean July temperatures range 12~18℃. Frost occurs from the end of August to early June on 280~300 d yr^{-1} (Tsedendash 1993). Annual precipitation at the weather station (at Shariin gol) was 256.23 mm, and mean annual temperature was 1.12℃ for the years 1996~2004. Dry periods occurred from 10 April to 30 July during these 9 yr; however, extreme dryness did not occur, which often takes place in the dry steppe and Gobi desert. Most precipitation falls during summer, from June to August. Livestock is traditionally kept in the study area because subtaiga forests border the steppe zone, which is crucial for the livestock of nomadic herder families, and the grazing capacity tends to be higher in the biome transition zone between forests and steppe.

In this study, 3 plots with ages ranging 10~65-yr representing young (P1), juvenile (P2), and old (P3) stands were selected for analysis. The dbh and height of all trees taller than 1.3 m were measured. Basic inventory information and the measured descriptive characteristics of the study plots are respectively shown in Tables 1 and 2. According to the history of forest management, the P1 and P2 stands were originally not forests. These stands resulted from an expansion of the forest area due to favorable site conditions and abundant parent material (seed sources and advanced regeneration). Only a few old trees grow around the P1 stand, while a large area of old forest can be found adjacent to the P2 stand. Since no records revealed that the forest stand of P3 resulted from forest expansion, the P3 stand should be an original forest.

All 3 stands were examined, and dbh and height were found to be negatively skewed (Table 2). This indicates that the diameter and height of the stands were negatively skewed or left-tailed distributed. In terms of the small

Table 1. Inventory information of the study site

Plot no.	Year of inventory	Plot size	Number of trees		Average age of plot trees (yr)
			by plot area	$/ha^{-1}$	
P1	2005	50 x 40 m	239	1195	10~15
P2	2005	40 x 40 m	250	1563	40~45
P3	2001	50 x 20 m	96	960	60~65

Table 2. Descriptive diameter at breast height (dbh) and height statistics of the study site

Plot no.	Variables	Mean	Standard deviation	Minimum~maximum	Skewness	Kurtosis
P1	dbh (cm)	16.10	6.03	2.50~30.00	-0.23	-0.75
	Height (m)	14.06	3.58	3.30~19.66	-0.94	0.21
P2	dbh (cm)	15.80	6.18	2.50~30.00	-0.17	-0.82
	Height (m)	11.88	3.25	6.19~15.02	-0.82	-0.91
P3	dbh (cm)	22.10	5.58	8.30~36.80	-0.02	-0.1
	Height (m)	15.89	2.58	6.11~21.62	-1.36	3.67

value (close to 0) of P3 dbh, the diameter distribution of the P3 stand was almost symmetrically tailed, which was the exception among the tested stands. The negative kurtosis of dbh showed that the forest distribution of the study sites was a little bit flatter than a normal distribution. The height of the P2 forest had a negative kurtosis, but the kurtosis of P1 and P3 was positive. This indicates that P1 and P3 heights had peaked distributions, while that of P2 was flat.

Modeling

Models

This study modeled dbh and height distributions of the forest plots with 3 functions, including the Burr, Dagum, and Johnson SBs provided by the software Easyfit 5.5 Professional. Both the PDF and cumulative density function (CDF) of the tested functions are presented.

Burr distribution

The Burr distribution (Zimmer and Burr 1963) has a flexible shape, and controllable scale and location, which makes it appealing for data fitting. It is sometimes considered to be an alternative to a normal distribution when data show a slightly positive skewness. For a random variable, x, such as dbh or height, with the boundary $\gamma \le x < +\infty$, the PDF of the Burr 4-parameter distribution is:

$$f(x) = \frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}}; \qquad (1)$$

where k and $\alpha > 0$ are the 2 shape parameters, $\beta > 0$ is the scale parameter, and γ is the location parameter. If $\gamma = 0$, then the distribution can be simplified to a 3-parameter one. The CDF of the Burr 4-parameter distribution of the random variable, $\boldsymbol{x}$, is:

$$F(x) = 1-\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{-k}. \qquad (2)$$

Dagum distribution

The Dagum distribution (Dagum 1977) is an inverse Burr distribution (Klugman et al. 1998). It is also known as the kappa distribution (Mielke 1973, Mielke and Johnson 1973) and extended Burr-III distribution (Shao et al. 2008). With the support random variable, x ($\gamma \le x < +\infty$), the PDF of the Dagum 4-parameter distribution is:

$$f(x) = \frac{\alpha k \left(\frac{x-\gamma}{\beta}\right)^{\alpha k-1}}{\beta\left(1+\left(\frac{x-\gamma}{\beta}\right)^{\alpha}\right)^{k+1}} \quad (\gamma \le x + \infty); \qquad (3)$$

where k and $\alpha > 0$ are the 2 shape parameters, $\beta > 0$ is the scale parameter, and γ is the location parameter. If $\gamma = 0$, then the distribution is a 3-parameter one. The CDF of the Dagum 4-parameter distribution is:

$$F(x) = \left(1+\left(\frac{x-\gamma}{\beta}\right)^{-\alpha}\right)^{-k}. \qquad (4)$$

Johnson SB distribution

The Johnson SB (1949) has been very commonly used in forest distributional studies (Hafley and Schreuder 1977) because of its flexibility of distributional form and its ability to equally well represent positively and negatively skewed distributions. Equations (5) and (6) show the PDF and CDF of the Johnson SB distribution for a random variable, $\boldsymbol{x}$. In the equations, ε and λ denote location and range parameters, δ and γ are 2 shape parameters, and Φ is the Laplace integral (Zhang et al. 2003).

$$f(x)=\left(\frac{\delta}{\sqrt{2\pi}}\right)\left(\frac{\lambda}{(x-\varepsilon)(\lambda+\varepsilon-x)}\right)\exp\left(-\frac{1}{2}\left(\gamma+\delta\ \ln\left(\frac{x-\varepsilon}{\lambda+\varepsilon-x}\right)\right)^2\right). \quad (5)$$

$$F(x)=\Phi\left(\gamma+\delta\ \ln\left(\frac{x-\varepsilon}{\lambda+\varepsilon-x}\right)\right). \quad (6)$$

Model fitting

In these PDFs, parameters can be estimated by different statistical methods such as maximum-likelihood (ML), moments, and percentiles. Parameters of the PDFs in this paper were estimated by the ML method using the distribution fitting software, Easyfit 5.5 Professional (MathWave Technologies 2013).

Model comparison

To evaluate the fitted PDF models, Kolmogorov-Smirnov (K-S), Anderson-Darling (A-D), and χ^2 tests were used for testing the goodness-of-fit. The K-S test (Kolmogorov 1933, Smirnov 1948) is a nonparametric test of the equality of a continuous, 1-dimensional probability distribution that can be used to compare a sample with a reference (or theoretical) probability distribution. It is based on the empirical CDF. The CDF is the probability that the variable takes a value of $\leq \boldsymbol{x}$. For continuous distributions, the CDF is expressed by equation (7):

$$F(x)=\int_{-\infty}^{x} f(t)dt. \quad (7)$$

So the theoretical CDF is displayed as a continuous curve. The empirical CDF (Eq. 8) is displayed as a stepped discontinuous line based on the number of bins:

$$F_n(x)=\frac{1}{n}*[\text{Number of observations} \leq x]. \quad (8)$$

The K-S statistic is based on the largest vertical difference between the theoretical and empirical CDFs. Critical K-S values used in this study were based on a table published in the statistical literature (D'Agostino and Stephens 1986). The A-D test (Anderson and Darling 1952) is also a nonparametric test of whether there is evidence that a given sample of data did not arise from a given probability distribution. It is more sensitive to the tails of a distribution than the K-S test. The χ^2 test (Chernoff and Lehmann 1954) is used for binned data and checks if sample data came from a specific distribution. The value of the test statistic depends on how the data is binned. The dbh and height data were grouped into intervals of equal probabilities. Each bin should contain at least 5 or more data points, otherwise no answer is available.

Hypothesis tests of dbh and height structure models were carried out by examining the *p* value that was associated with a goodness-of-fit statistic. When the *p* value was less than a predefined critical value or a significant probability level, the null hypothesis was rejected, and it was concluded that the data did not come from the specified distribution.

RESULTS

Figure 1 demonstrates the tallied frequency of the observed dbh and height of the studied forests. The stem dbh and height were respectively grouped into 2-cm and 1-m widths. The derived PDF distribution models of each site by Burr, Dagum, and Johnson methods are also demonstrated. Table 3 shows the corresponding parameters of the derived distribution models in Fig. 1. According to the test results of the goodness-of-fit shown in Table 4, the Dagum and Burr distribution functions are suitable for representing the dbh structure of Scots pine forests because the K-S, A-D, and χ^2 statistics all agreed that the dbh distribution models were acceptable for young, juvenile, and old forests. The Johnson

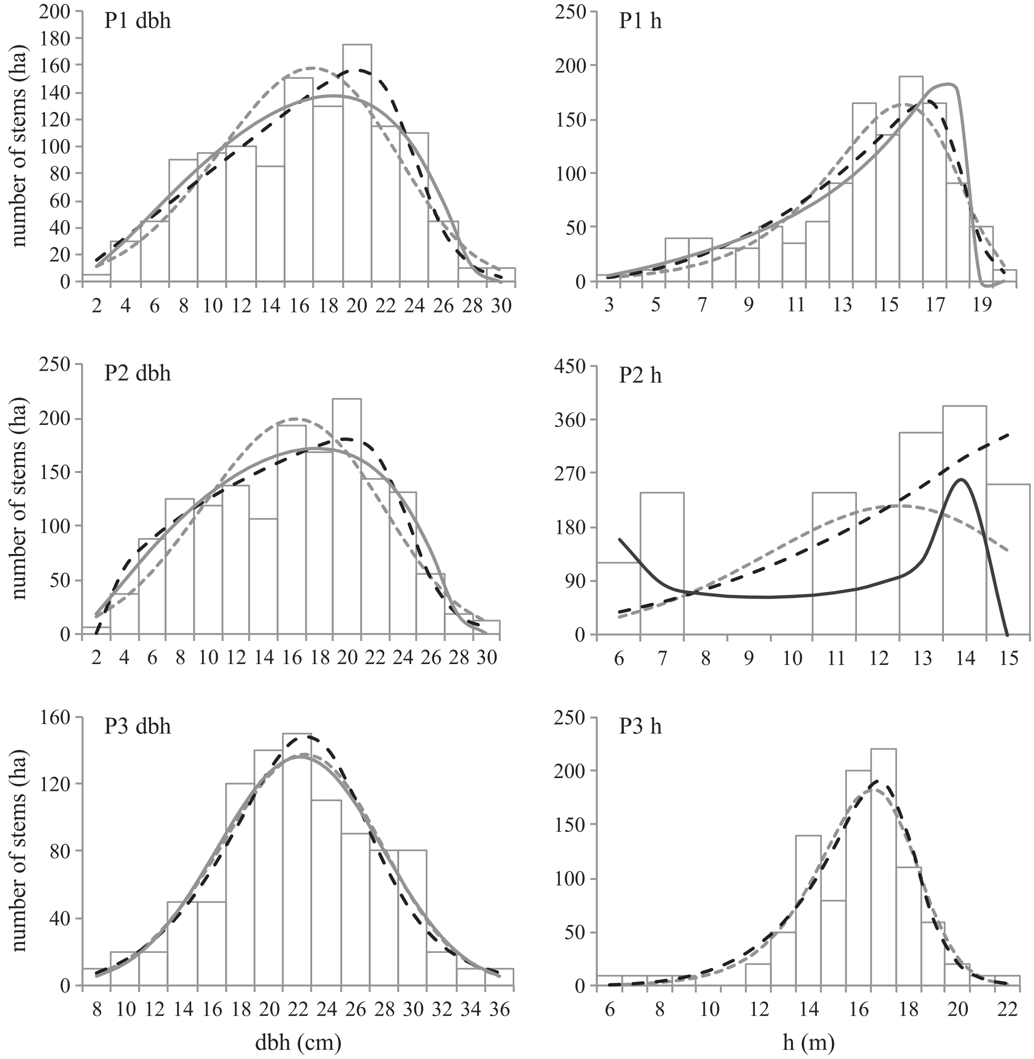

Fig. 1. Diameter at breast height (dbh) (left) and height (right) model comparisons for pine forests in study plots P1~P3. The histogram represents the observed distribution, and the short dashed line (Burr), long dashed line (Dagum), and solid line (Johnson SB) show the estimated distributions.

SB distribution function was acceptable only for old forests that had a normal dbh distribution.

Concerning the derived height distribution models, Dagum and Burr distribution functions were also able to accurately represent height distributions of young and old Scots pine forests. These 2 distribution functions failed to express the juvenile stand, which was probably due to the height of those stems in the studied juvenile stand being discretely or irregularly distributed. By inspecting the distribution structure, it seems more like a 2-peak distribution. In other words, a mixed distribution model (Tsogt and Lin 2013) was probably suitable for this case. The Johnson SB distribution function was not able to fit height observations of the Scots

Table 3. Parameter estimates of the diameter at breast height (dbh) and height distribution models for pine forests, P1~P3

Plot no.	dbh			Height		
	Burr	Dagum	Johnson $_{SB}$	Burr	Dagum	Johnson $_{SB}$
P1	$k = 96.872$	$k = 0.12207$	$\gamma = -0.33446$	$k = 3422.4$	$k = 0.10296$	$\gamma = -1.0667$
	$\alpha = 4.3718$	$\alpha = 16.329$	$\delta = 1.0467$	$\alpha = 489.29$	$\alpha = 32.122$	$\delta = 0.83265$
	$\beta = 70.738$	$\beta = 24.025$	$\lambda = 30.387$	$\beta = 1328.3$	$\beta = 18.134$	$\lambda = 18.118$
	$\gamma = -6.4707$	$\gamma = 0$	$\xi = -1.0791$	$\gamma = -1290.7$	$\gamma = 0$	$\xi = 0.79728$
P2	$k = 108.79$	$k = 0.09438$	$\gamma = -0.23789$	$k = 1045.9$	$k = 0.00778$	$\gamma = -0.55124$
	$\alpha = 3.8956$	$\alpha = 15.688$	$\delta = 0.99499$	$\alpha = 4.7765$	$\alpha = 442.93$	$\delta = 0.27967$
	$\beta = 76.963$	$\beta = 22.283$	$\lambda = 29.794$	$\beta = 56.121$	$\beta = 15.101$	$\lambda = 9.1613$
	$\gamma = -5.0039$	$\gamma = 2.3031$	$\xi = -0.52346$	$\gamma = 0$	$\gamma = 0$	$\xi = 5.5655$
P3	$k = 8.253$	$k = 0.43323$	$\gamma = -0.30612$	$k = 3.0013$	$k = 0.398$	$\gamma = 0.51806$
	$\alpha = 4.7694$	$\alpha = 9.8894$	$\delta = 4.3887$	$\alpha = 1.4860\text{x}10^6$	$\alpha = 147.07$	$\delta = 3.5092$
	$\beta = 36.962$	$\beta = 25.513$	$\lambda = 99.144$	$\beta = 2.4671\text{x}10^6$	$\beta = 123.03$	$\lambda = 84.116$
	$\gamma = 0$	$\gamma = 0$	$\xi = -29.188$	$\gamma = -2.4671\text{x}10^6$	$\gamma = -105.52$	$\xi = -16.76$

pine forests. The Weibull distribution function was found (results not shown) to be able to express the dbh and height structure of the stands, but the test of goodness-of-fit showed that the results were identical to the derived Burr distribution models.

DISCUSSION

Distribution features of the derived dbh models

By visual inspection of the histogram shown in Fig. 1, one can see that young and juvenile forests had peaks in the 20-cm dbh class. The derived Burr and Johnson SB models peaked at the 16-cm position, which was somewhat distant from the observed peak location. In contrast, the curve of the derived Dagum model peaked at the same location as the observed one. The dbh histogram of the old forest was almost identical to a normal distribution. In this case, the curves of the derived Dagum, Burr, and Johnson SB models accurately peaked at the location of the observed peak. This indicated that the Dagum function had a better ability to obtain the specific curve feature which is the most clumped dbh class. Peaks estimated by the Burr models were almost identical to the average dbh position of the observed histogram. This can be seen from Table 2. The derived Johnson SB model peaked at a position covering the dominant DBH classes, which made its curve a little bit flat. This was more evident in plot 2 of juvenile forests in Fig. 1.

The P1 and P2 dbh distributions were identical, even though their geographical locations and ages differed. According to their dbh distribution shape, their regeneration and growth patterns were the same. In P1 and P2, most stems were established shortly after a disturbance, and this still left enough growing space for stems to become established later, in 1960~1965 (P2) and 1990~1995 (P1) (Tsogt et al. 2008). During 5 yr, many seedlings successfully grew and survived after regeneration; those are 10-yr old in P1 and 40-yr old in P2. They are evident on the P1 and P2 dbh distributions of Fig. 1, where plateaus are on the left side of the peak. Left-tailed dbh distributions indicate that growth space is still sufficient (in free growth situations) for

Table 4. Goodness-of-fit and ranking (rank in parentheses) of the Burr, Dagum, and Johnson SB distributions for the empirical diameter at breast height (dbh) and height distributions as measured by maximum-likelihood estimation criterion (α = 0.05)

Plot no. Structure	Distribution	Kolmogorov Smirnov statistic	Anderson Darling Statistic	Chi-squared statistic
P1 dbh	Critical value	0.08784	2.5018	14.067
	Burr (4P)	0.06589 (3)	1.1113 (2)	7.9501 (2)
	Dagum (3P)	0.04047 (1)	0.40807 (1)	7.7654 (1)
	Johnson S_B	0.04462 (2)	4.3858 (*)	-
P1 Height	Critical value	0.08784	2.5018	14.067
	Burr (4P)	0.07234 (2)	1.4021 (2)	13.7770 (2)
	Dagum (3P)	0.06231 (1)	0.95645 (1)	8.4542 (1)
	Johnson S_B	0.07286 (3)	38.763 (*)	-
P2 dbh	Critical value	0.08589	2.5018	14.067
	Burr (4P)	0.06888 (3)	1.3299 (2)	9.136 (2)
	Dagum (4P)	0.03904 (1)	0.26976 (1)	4.6169 (1)
	Johnson S_B	0.04521 (2)	4.4621 (*)	-
P2 Height	Critical value	0.08589	2.5018	7.8147
	Burr (3P)	0.25849 (*)	22.676 (*)	30.646 (*)
	Dagum (3P)	0.26938 (*)	25.977 (*)	14.728 (*)
	Johnson S_B	0.16567 (*)	142.13 (*)	-
P3 dbh	Critical value	0.13675	2.5018	12.592
	Burr (3P)	0.04506 (2)	0.20612 (2)	3.8621 (3)
	Dagum (3P)	0.05222 (3)	0.24601 (3)	2.7432 (2)
	Johnson S_B	0.04361 (1)	0.18449 (1)	0.9937 (1)
P3 Height	Critical value	0.13675	2.5018	12.592
	Burr (4P)	0.04699 (2)	0.3492 (2)	1.8253 (2)
	Dagum (4P)	0.04166 (1)	0.22303 (1)	1.6842 (1)
	Johnson S_B	No fit		

* Assumption rejected at α = 0.05.

major trees which occupy the main canopy of the stand, and the dbh distribution structure will not change until the forest reaches the maximum capacity of the stem number and mean size ratio. Once a forest reaches the maximum tree density-size ratio, individual tree growth can continue only if the number of individuals is reduced (Yoda et al. 1963, Kimmins 2004). Thus, the forest dbh distribution structure may change with different distribution shapes depending on the size of the tree stems removed from the stand (O'Hara and Gersonde 2004). The P3 dbh distribution indicates that some trees dominated the major population, while some were suppressed. In theory, dominant trees are located in more-favorable microclimatic conditions than suppressed trees or they may have just inherited good genetic materials (Oliver and Larson 1996). Either way, dominant trees grow bigger, faster, and stronger, while other trees grow more slowly and are suppressed. However, the dbh structure of the main population was still quite normal in the P3 forest.

Distribution features of the derived height models

As can be seen, height distributions of P1 and P3 were continuous with long left tails and outstanding peaks at the larger end of the curves, while that of P2 was discrete with 2 separate distinct groups (2 gaps exist at 8~10 and 12 m). It was also evident that the height growth of the P2 stand was worse than those of P1 and P3. This demonstrated that the P1 and P3 stands were growing without adverse influences or effective disturbances, but on the contrary, the P2 stand appeared to have been significantly disturbed. Another consideration is that tree height repression occurs first in short trees with small trees crowns. In our case, those were 6- and 7-m-high trees in the P2 distribution which were ~40-yr-old stems according to the graphical analysis in Fig. 1. This can occur in individual trees as they become suppressed or in entire stands as they approach stagnation (Eversol 1955). This explains why the average height of P2 was shorter than that of P1. As a result, the Burr, Dagum, and Johnson functions were able to extract suitable distribution models for the non-disturbed stands, but failed to derive a meaningful height structure of the discretely distributed stand.

According to Oliver and Larson (1996), trees with close neighbors on both sides maintain small live crowns, and their height growth eventually slows. When an adjacent tree has a wider spacing on the far side, it maintains rapid height growth. The canopy of the P2 stand was much closer than those of P1 and P3, this was probably why the P2 stand showed a smaller height at age compared to the P1 and P3 stands.

There are many factors that might influence the structure of forest stands. There are inherent factors (such as aging and genetics), physical factors (such as site properties), natural disturbances (such as fire, wind, insects, and disease), and management practices (such as thinning and logging) (Oliver and Larson 1996). In even-aged stands, competition for light, water, and nutrients depends largely on the number of stems per unit area. Stand canopy closure might become pronouncedly heterogeneous due to a combination of environmental and genetic factors. Some trees became dominant while others became suppressed over time (Barnes et al. 1998). Any changes in these factors will cause the continuous forest distribution to break down, creating a discrete or irregular distribution. In this situation, a better suggestion for modeling of the forest structure is to first diagnose the observed histogram and then fit the distribution using a mixed model.

CONCLUSIONS

The dbh and height distributions of naturally generated, even-aged young pine forests are left-tailed in Western Khentey, Mongolia. Few distribution functions have the ability to simulate left-skewed distributions. The Burr, Dagum, and Johnson SB PDFs can theoretically describe both left- and right-skewed distributions. However, our study showed that the Dagum distribution was superior according to the statistics of the model goodness-of-fit test. The Burr PDF was generally good enough to model the diameter and height distributions. The Johnson SB was found to be the best only for the case of a normal distribution. In fact, the Johnson SB is good at extracting heavily tailed or bounded tail shapes which were not found in this study.

dbh and height distribution models can be employed to predict the number of stems of various dbh and height classes of a forest stand up to 65 years of age. In addition, the height distribution of a forest stand should be considered and determined simultaneously for a better explanation of the forest structure and also for better estimates of forest volume stocks. For example, the dbh distribution shapes of P1 and P2 were almost identical; they could be considered to have same structure, while their height structures obviously greatly differed. The height distribution of P2 revealed the degree of seriousness the competition for resources among tree stems in the stand was. Moreover, although the dbh distribution of the old forest stand showed a normal distribution, its height distribution contrarily demonstrated that some trees were constantly suppressed by dominant trees. Without an analysis of the height distributions, we could not have figured out much information about the formation of the forest structure. This kind of information is vital for forest management planning and thinning or an understanding of how structures of natural forests behave due to variations of driving factors (stem density and other environment factors).

For actual applications, it is suggested that the Burr function is suitable for deriving the distribution as its peak tends to average classes of forest diameter and height structure, while the Dagum function is better for finding the dominant size class. The Johnson SB function is suggested for drawing a flattened distribution shape where dominant size classes are clumped together or size classes are equally distributed along the range of distribution (Tsogt and Lin 2011c).

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