Research note

Taper Modeling on *Calocedrus formosana* Plantations in Lienhuachih, Central Taiwan

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【Summary】

Taper is used to measure the rate of decrease in stem diameter from the bottom upwardly. Taper equations express the expected stem diameter outside or inside the bark, as a function of the height above ground level, total tree height, and diameter at breast height. Four tapering modeling approaches were used to estimate the stem diameter at a given height above the stump in *Calocedrus formosana* plantations and compare their accuracy and precision. The results indicated that while there was little difference in describing the overall tapering performance, based on 3 criteria considered simultaneously, the variable-form stem profile was the best at describing the taper of the total tree bole, followed by 3-segmented polynomials. The mean relative biases for these models on the fitted trees were < 6%, except for the polynomial form with a higher-order approach. The butt swell (segment from 0.3 to 1.3 m in height) is a part of the stem that is the most difficult to predict in tapering modeling for all models used. However, the precision and accuracy in the prediction of the tree butt swell can be impressively improved using variable-form taper models.

Key words: stem profile, variable form, segmented polynomial.

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研究簡報

蓮華池地區台灣肖楠人工林林木尖削度模式之建立

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摘　要

林木尖削度模式又稱為樹幹剖面模式(stem profile)，是描述樹幹連皮或去皮直徑隨著某特定高度之變化情形。尖削度可以計算一株樹在不同利用標準下之木材利用材積。因此，對木材工業而言，良好之尖削度模式可以在不同標準規格下提供準確有效之木材利用率。一般言之，尖削度模式種類甚多其配置之方式亦不同。本研究使用高階多項氏、二段式多項式、三段式多項式和變動形數模式等4種模式，探討蓮華池地區台灣肖楠人工林木尖削度模式之建立，並就各模式之推估能力進行比較。研究結果顯示雖然就總體之推估能力(含偏差與精密度)言之，各模式間之差異不大，以變動形數模式最佳，但描述各不同段位之能力則有明顯之差別。以從根株至10%樹高高度之部位觀之，變動形數模式之預測能力最佳，其次為三段式多項式。由於變動形數模式和三段式多項式能較有效描述各段位之尖削度變化，不但其總體之偏差百分比小於6%，其幹材較低部位之偏差亦為最低，因此，變動形數模式和三段式多項式是為比較理想之尖削度模式。

關鍵詞：樹幹剖面模式，變動形數模式，分段式多項式。

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A tree taper model is a mathematical representation of the stem shape, or the change in diameter along the stem. Taper models are widely used in forestry to estimate stem volume and other parameters of stand trees, and are particularly useful for computing the top diameter limit for any utilization standard (Westfall and Scott 2010). Tree taper measurements contain discrete pair-wise height and relative diameter measurements at certain locations along the length of the tree (Avery and Burkhart 1994, Ramazan et al. 2011).

Many approaches have been developed for producing taper equations. These include early taper equations based on simple functions (e.g., Kozak et al. 1969, Ormeord 1973, Amidon 1984). Max and Burkhart (1976) introduced segmented polynomials, in which a tree is divided into 3 distinct sections using submodels to describe each section. Later, Kozak (1988), Newnham (1992), and Sergio et al. (2013) applied variable-form or variable-exponent stem profile models using the exponent changes from the base to the tip of the tree to account for differing shapes along the stem profile. Moreover, Rustagi and Loveless (1991), Zhang et al. (2002), and Jordan et al. (2005) used compatible taper models in which the mathematical relationships are compatible among taper, total volume, and merchantable volume.

Wang et al. (2007) developed taper equations for Taiwania (*Taiwania cryptomerioides*) plantations in the Liukuei area. In their study, they used 5 approaches to build taper models and compared the accuracy and precision of taper predictions among the 5 approaches. Their results showed that the trigonometric taper modeling approach was the worst in describing the taper of the entire tree bole, followed by the sigmoid form approach. On the other hand, segmented polynomials, variable-form stem profiles and polynomial forms with a higher-order approach were preferred for describing the taper in the entire stem.

Taiwan cedar (*Calocedrus formosana* (Florin) is a very important native species on Taiwan plantations. Taiwan cedar has a good wood quality for manufacturing with a very high economic value and is ranked as one of the 5 most valuable species of conifers in Taiwan (Liu et al. 1988). The 4th forest inventory estimated 1025 ha of Taiwan cedar plantations in Taiwan by the end of 2014 (TFB 2016). While several studies of Taiwan cedar have been conducted in the past, e.g., thinning effect (Hung et al. 1982, Chiu et al. 2014), growth and wood properties (Liu and Wu 1986), and flowering promotion and seedlings (Chung et al. 2001), no work on the tree stem profile in Taiwan cedar plantations has been done. Therefore, the purpose of this study was to develop a taper model and compare the accuracy and precision of taper predictions among several approaches.

Based on the results from Wang et al.'s (2007) study, segmented polynomials, variable-form stem profile and polynomial form with a higher order approach were adopted in this study. The associated mathematical expressions of the formulation for each type used in this study are given in Table 1.

The precision and accuracy of all tree profile prediction systems were determined by the following criteria in terms of diameters at different heights:

1) average bias in cm = $\Sigma(d_i - d_{ihat}) / n$;
2) average bias in percentage = $\Sigma((d_i - d_{ihat}) / d_i \times 100) / n$;
3) average absolute bias in cm = $\Sigma abs((d_i - d_{ihat}) / n)$;
4) standard error of the estimate (SEE) in cm = $Sqrt (\Sigma(d_i - d_{ihat})^2 / (n - m - 1))$; and
5) Mean squared error (MSE) = $Bias^2 + Variance$;

where d_{ihat} is predicted at height h_i and d_i is the actual measurement at point i on the bole; m is the number of parameters in the model, and n is the number of points in a specified region of relative height, say 0.1~0.2 of total height.

Data for this study came from Taiwan cedar plantations in the Lienhuachih Experimental Forest of the Taiwan Forestry Research Institute. Tree tapers (tree forms) can be obtained through several means, primarily direct measurement, felled tree stem analysis, measurement with dendrometers, and other optical methods (Larsen, 2006). In this study, we used stem analysis and sample trees came from plantations of Taiwan cedar in Compartment 2. Single-stemmed trees without broken tops representing a variety of tree sizes in diameter at breast height (DBH) and in height were selected for felling. Diameters outside the bark were measured at ground stump height (0.3 m), breast height (1.3 m) and every 1-m height interval above breast height. The total height and crown base height were recorded as well for each individual tree. Moreover, the boles of trees were cut into sections. Discs at stump height, breast height and 2-m intervals in height above 1.3 m for each tree were carried into the laboratory for a stem analysis. Due to the consideration of sample size, all individual measurements of diameter / height for trees were used for fitting purposes.

Distributions of DBH and tree height on sample trees aged about 50 yr are shown in Table 2. The estimated coefficients and their standard errors for all models are presented in Table 3.

The ratio of DBH to total height (H) is an indicator that is widely used to estimate the form or the slenderness of a tree bole (Hann et al. 1987). The effect of DBH/H on

Table 1. Summary of methods adopted in this study

Methods	Equations*
Polynomial form with a higher order approach (Bruce *et al* (1968)) model 1	$d^2 = DBH^2 \times \{ b_1 \times x^{1.5} \times (10^{-1}) + b_2 \times (x^{1.5} - x^3) \times DBH \times (10^{-2})$ $+ b_3 \times (x^{1.5} - x^{3)}\ HT \times (10^{-3}) + b_4 \times (x^{1.5} - x^{32}) \times HT \times DBH \times (10^{-5})$ $+ b_5 \times (x^{1.5} - x^{32}) \times HT^{1/2} \times (10^{-3}) + b_6 \times (x^{1.5} - x^{40}) \times HT^2 \times (10^{-6})\}$ where $x = (HT - h) / (HT - 1.3)$ It meets the tip pass requirement.
Segmented polynomials with one joint point of two submodels (Max and Burkhart (1976)) model 2	$d^2 / DBH^2 = b_1 \times (h / HT - 1) + b_2 \times [(h / HT)^2 - 1]$ $+ b_3 \times (\alpha_1 - h / HT)^2 \times I_+(\alpha_1 - h / HT)$ Where α_1 : a joint point of the submodels $I_+(\alpha - h / HT)$: a dummy variable with a value 1 When $\alpha_1 \geqq h / HT$, of 0, otherwise It meets the tip pass requirement
Segmented polynomials with two joint points of three submodels(Max and Burkhart (1976)) model 3	$d^2 / DBH^2 = b_1 \times (h / HT - 1) + b_2 \times [(h / HT)^2 - 1]$ $+ b_3 \times [(\alpha_1 - h / HT)^2 \times I_+ (\alpha_1 - h / HT)] + b_4 \times [(\alpha_2 - h / HT)^2$ $\times I_+(\alpha_2 - h / HT)]$ where α_1, α_2 : two joint points of the submodels $I_+(\alpha_1 - h / HT)$: a dummy variable with a value of 1 when $\alpha_1 \geqq h / HT$, of 0, otherwise $I_+(\alpha_2 - h / HT)$: a dummy variable with a value of 1 When $\alpha_2 \geqq h / HT$, of 0, otherwise It meets the tip pass requirement.
Variable-form stem profile models (Kozak (1988) model 4	$d = b_1 \times DBH^{b2} \times b_3^{DBH} \times X^C$ where X : $(1 - \mathrm{sqrt}(Z)) / (1 - \mathrm{sqrt}(I))$ Z : h / HT I : location of the inflection point C : $b_4 \times Z^2 + b_5 \times \ln(Z + 0.001) + b_6 \times \mathrm{sqrt}(Z) + b_7 \times e^Z$ $+ b_8 \times (DBH/H)$ It meets the tip pass requirement.

* Notation: d is diameter outside / inside bark (cm) at a specific height h from ground; DBH is diameter outside bark at breast height (cm); h is height at a specific point on the bole (m) from ground; HT is the total tree height (m); b_i, i = 1, 8 α_1, α_2 are parameters to be estimated.

the taper based on observations of 3 sample trees showed that slender trees (i.e., a lower DBH/H) tapered more quickly than trees with a high DBH / H (Fig. 1).

The overall shape of the tree stem-profile is shown in Fig. 2. It shows that the inflection point (the point where the tree form changes from neiloid to paraboloid) roughly occurred at 0.08 of the relative height.

As it may reveal potentially different capabilities inherent in describing tapering on the portion of the stem for the models used, the comparisons of prediction power among models should focus not only on the entire stem but also on individual parts of the stem (Biging 1984, Bailey 1994, Li et al. 2012). In this study, a relative height with 6 levels (e.g., 0.3~1.3 m, 1.3 m~20, 20~40, 40~60, 60~80, and 80~100%) of the total height was used to identify segments of the stem above the DBH.

Comparisons among all models indicated that for the overall tree bole, a small average

Table 2. Distribution of sample trees by diameter breast hight (DBH; cm) and total height (HT; m)

DBH / HT	16~18	18~20	20~22	22~24	24~26	26~28	28~30	>30
15~20	1	1						
20~25			1		1			
25~30		1	1	1	2			
30~35			2		1	1		
35~40						1		
40~45					1			
45~50				1		1	1	
50~55				1	1		1	
total	1	2	4	3	6	3	2	

Table 3. Estimated coefficients and their standard errors (in parenthesis) of the diameter outside bark for all models based on fitted data

model	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	α_1	α_2
1	10.287	2.162	-10.732	-35.106	12.326	94.185				
	(0.092)	(0.255)	(3.329)	(2.281)	(5.883)	(41.145)				
2	-1.408	0.371	47.933						0.1	
	(0.050)	(0.038)	(2.964)							
3	-0.702	0.072	0.451	52.814					0.1	0.8
	(0.702)	(0.408)	(0.473)	(3.112)						
4	0.847	0.968	1.003	0.605	-0.177	0.463	-0.119	-0.008		
	(0.227)	(0.104)	(0.003)	(0.272)	(0.064)	(0.645)	(0.341)	(0.015)		

model 1 Polynomial form with a higher order approach :
model 2 Segmented polynomials with one joint point of two submodels;
model 3 Segmented polynomials with two joint points of three submodels;
model 4 Variable-form stem profile models;

bias in diameter was detected (Table 4). In relative terms, the mean biases in percentage were < 6% for all equations except equation 1. As judged by the first 3 criteria considered simultaneously, the variable-form stem profile performed best to describe the taper in the total stem (Table 4). As a biased estimator with a small variance may be preferable to an unbiased estimator with a large variance, the MSE was also used to evaluate the diameter estimators (Devore 1982, Coble and Wiant Jr. 2000). The rank of models based on the MSE is listed in Table 4.

As to the power of the variable exponent in depicting the taper profile for different segments, the variable-form taper model best fit in the entire profile (Table 4) and also in different segments of stems (Table 5). In segmented polynomials with submodel taper models, 2 number of joints were fitted to the data. This study showed slight improvement in the predictive ability of the 2-joint submodel over the 1-joint submodel over most segments. This is because formulating a taper model by splining together 3 polynomials could describe the profile of one of the 3 segments for each polynomial (Valentine and Gregoire 2001).

The exponent from ground to top for different sizes of *Calocedrus formosana* plantations in this study indicated that the exponent

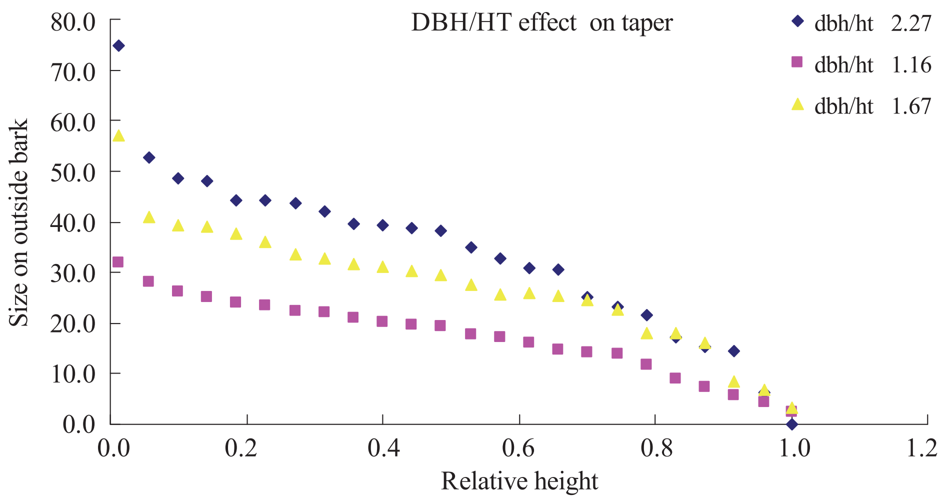

Fig. 1. Effects of diameter at breast height (DBH) / total height (H) on taper based on observations of 3 sample trees.

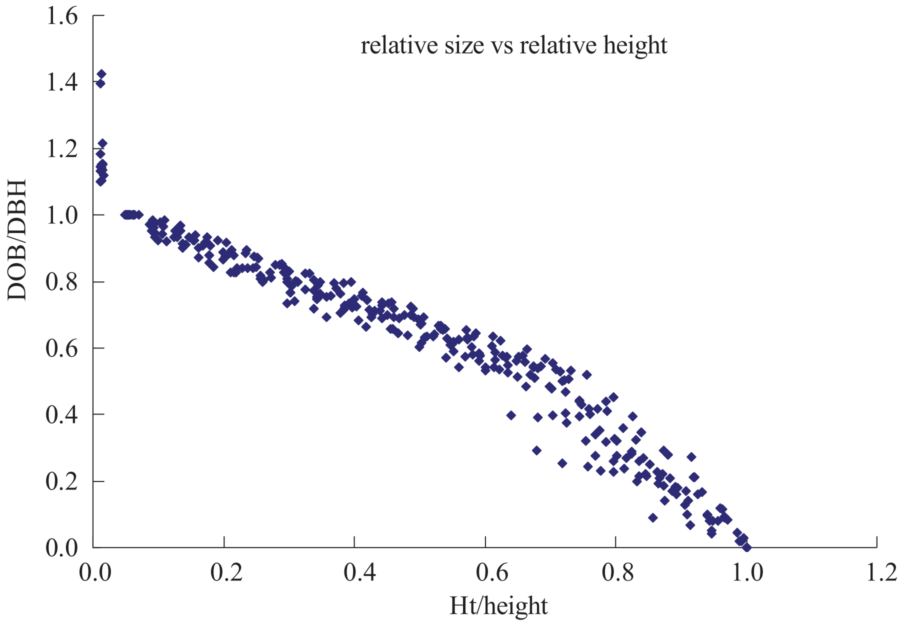

Fig. 2. Plot of the relative size vs relative height for fitting data. DOB is the diameter at outside bark at a given height (h), DBH is the outside bark at breast height, HT is the total tree height.

value was variable at different relative heights above the ground (Fig. 3). The general trend of the exponent was similar for different sizes of trees. It can be observed that the inflection point seemed to be quite constant (almost 0.25) regardless of the tree size.

In conclusion, although there was still a slight bias near ground level (of < 0.6%), for the variable exponent models and the 3-segmented polynomial models, their pre-

Table 4. Comparison of the bias and standard error of the diameter outside bark among models for the total tree bole in fitted data

Model equation	Bias			Standard error (cm)	MSE (cm^2)	Rank
	Average (cm)	Percentage average (%)	Absolute average (cm)			
1	-0.34	-13.7	1.22	1.84	3.35	4
2	-0.05	-5.42	1.05	1.59	2.50	3
3	0	-3.13	1.06	1.49	2.17	2
4	-0.10	-5.52	1.00	1.38	1.89	1

MSE, mean squared error.
model 1 Polynomial form with a higher order approach :
model 2 Segmented polynomials with one joint point of two submodels;
model 3 Segmented polynomials with two joint points of three submodels;
model 4 Variable-form stem profile models;

Table 5. Average biases of the diameter outside bark in percentage (%) from the ground to the top for all models in fitting data

Height from the ground	Sample size	Model equation			
		1	2	3	4
0.3~1.3 m	21	-4.42	-0.42	-0.55	0.07
1.3 m~0.1 H	34	-0.50	-1.49	-0.71	-0.36
0.1 H~0.2 H	46	3.02	0.72	1.13	1.35
0.2 H~0.4 H	98	0.67	-0.49	-1.22	-1.53
0.4 H~0.6 H	98	-0.81	1.08	-0.22	-0.10
0.6 H~0.8 H	98	-2.45	-6.25	3.17	-1.90
0.8 H~1.0 H	106	-29.28	-23.31	-21.5	-12.66

H, total tree height.
model 1 Polynomial form with a higher order approach :
model 2 Segmented polynomials with one joint point of two submodels;
model 3 Segmented polynomials with two joint points of three submodels;
model 4 Variable-form stem profile models;

diction of diameters outside the bark had < 3% in bias over most of the lengths of the trees. Therefore, these 2 tree profile prediction systems describe the tree shape more realistically than any other taper system we tested.

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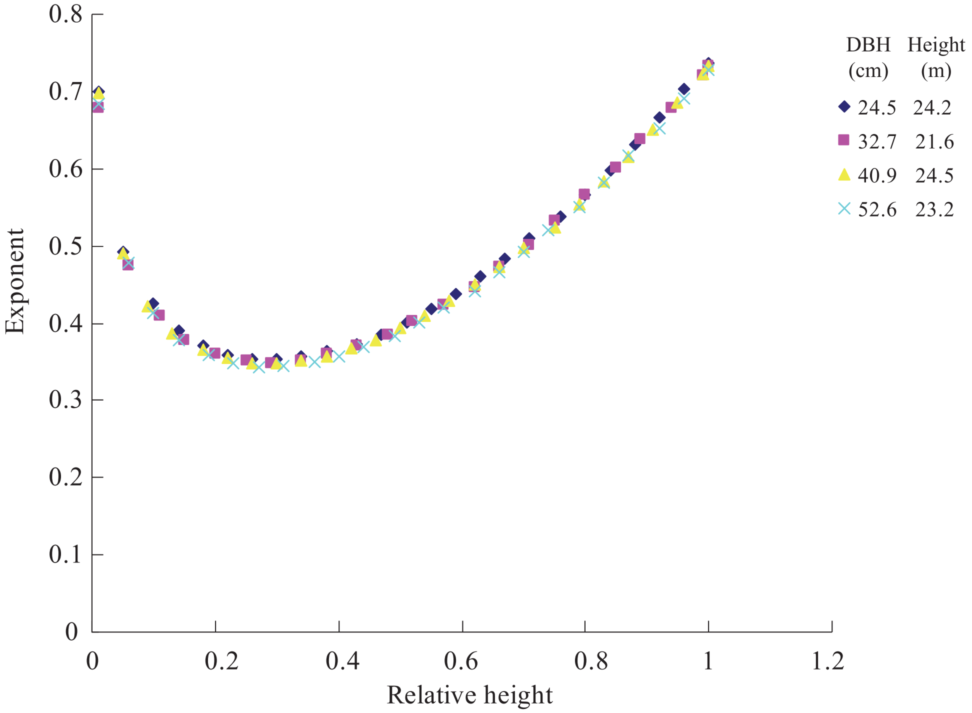

Fig. 3. Change in the exponent for *Calocedrus formosana* plantations for 4 trees.

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