研究報告

應用不同結構樹高曲線式模擬臺灣杉人工林之效果評估

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摘　要

樹高曲線式(height-diameter (H-D) model)係採用胸高直徑(diameter at breast height (DBH))推估樹高(tree height (H))的重要工具，然而H之模擬效果會隨著H-D model的結構而改變。本研究旨在探討模式結構對H-D model模擬表現之影響。研究區域位於臺灣中部地區惠蓀林場之臺灣杉(*Taiwania cryptomerioides* Hayata)人工林林分，共獲104株具DBH與H之單木資料。本研究採用不同種模式型態之H-D model進行建模，採用residual sum of squares (RSS)、root mean square error (RMSE)、Akaike information criterion (AIC)及relative rank (R-rank)等指標評估模式。並以成對樣本*t*-test (paired sample *t*-test)及二因子變異數分析(two-way analysis of variance (ANOVA))分析模式模擬之效果。結果顯示，在所有模式中$H = a + bD + cD^2 + d \log D$表現最佳。而非線性模式方面，約束模式通過原點可提升模擬效果；然而在線性模式方面，3及4參數模式模擬結果較2參數為佳。比較2種模式型態在參數間的模擬效果，非線性模式在2參數結果較佳，而在3及4參數則與線性模式效果相同。

關鍵詞：約束模式、惠蓀林場、參數數目、模式型態、臺灣杉。

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Research Paper

Assessing Prediction Effects among Height-Diameter Models with Varied Structures for a Taiwania (*Taiwania cryptomerioides* Hayata) Plantation

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【Summary】

The height-diameter (H-D) model is an important tool for predicting tree height (H) based on the diameter at breast height (DBH). However, the performance of the H-D model varies with the model structure. The purpose of this study was to examine the performances of H-D models with various model structures. The research site was located in central Taiwan. Data were collected from a Taiwania (*Taiwania cryptomerioides* Hayata) plantation at the Huisun Forest Station, and in total, the DBH and H of 104 individual trees were obtained. We adopted various H-D models with different structures to establish the models. The residual sum of squares (RSS), root mean square error (RMSE), Akaike information criterion (AIC), and relative ranking (R-rank) performance criteria were employed as criteria. A paired sample *t*-test and two-way analysis of variance (ANOVA) were used to assess model performances. Results showed that $H = a + bD + cD^2 + d \log D$ stood out among all models. Nonlinear models had better performances when they were constrained to pass through the origin. In linear models, the performances of 3- and 4-parameter models were better than those of 2-parameter models. In a comparison of the number of parameters between models, nonlinear models performed better than linear models at the 2-parameter level due to large biases in the linear models.

Key words: constrained model, Huisun Forest Station, number of parameters, model type, Taiwania.

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緒 言

樹高(tree height, H)與胸高直徑(diameter at breast height, DBH)為立木重要的性狀值(Clutter et al. 1983, Avery and Burkhart 1994)，可用於探討林木的木材品質(Chou et al. 2002)，並用以計算單木材積，擴及推估森林蓄積量、生物量與碳貯存量；林分結構亦常採用此兩性狀值為基礎進行模擬與描述(Sharma and Breidenbac 2015, Yen et al. 2020)。然而，H的調查相較於DBH不容易且需花費較多時間，特別在山區易受地形及周遭林木遮蔽的環境，此也增加山區測計H的難度(Colbert et al. 2002, Wang et al. 2012)。在林木生長收穫領域，為能解決上述的問題，研究人員常採用DBH與H的關聯性建立樹高曲線式(height-diameter (H-D) model)，即以DBH為自變數，H為因變數($H = f\,(DBH)$)，方便森林資源調查人員推估H，而此種方式也廣泛地應

用在森林測計領域，尤其是在人工林的應用，H-D model被視為推估森林蓄積量的重要工具(Trorey 1932, Meyer 1936, Huang et al. 2015)。

H-D model具有多種形態，可應用於不同地區與樹種(Dorado et al. 2006)，就模式的結構而言，可分為線性及非線性2種型式(Curtis 1967, Guimaraes et al. 2009)。一般而言，線性型H-D model求解與操作較為方便，但模式模擬常受限於線性型態的特性(Osman et al. 2013a)，相對於非線性型H-D model因曲線較具彈性，可更有效描述H與DBH之間的關係，唯在求解與操作方面較線性模式不易(Mensha et al. 2018)。除模式型態外，模式的參數亦為影響H-D model的重要因子之一(Liu et al. 2017)，依據模式參數數目，H-D model又可分為2參數、3參數或多參數模型(Lebedev and Kuzmichev 2020)，一般隨參數的增加，其模擬的效果也會隨之提升，同時模式之求解難度也會增加(Mehtätalo et al. 2015)。因此，建立H-D model時必須考量模式的多元特性，選擇適用的模式進行推估(Ahmadi et al. 2013)。

森林資源調查作業常採用H-D model為工具進行H的推估，以往H-D model大多著重於生長收穫之應用層面，較少探討H-D model在理論層面的特性。近年為達森林永續經營及林業振興等重要施政目標，國內造林多選用兼具生態及經濟價值的本土樹種，其中臺灣杉(*Taiwania cryptomerioides* Hayata)為近年造林最廣泛的樹種之一(Qiu et al. 2015, TFB 2020)，具生長快速、木材品質優良等多項優點(Hung 1974)，本研究以惠蓀林場第3林班臺灣杉人工林為例，探討不同型態的H-D model在臺灣杉人工林之適用性，本次研究目的為(1)建立臺灣杉人工林之H-D model；(2)比較模式結構與模擬效果之關係；及(3)探討參數數目對H-D model模擬效果之影響。

材料與方法

一、研究區域

本研究位於臺灣中部地區南投縣仁愛鄉境內，試驗地點隸屬於國立中興大學實驗林管理處所轄之惠蓀林場第3林班第278號造林地臺灣杉人工林(Fig. 1)，為瞭解林分發展特性，本研究於2017年1月在此造林地設置4個0.05 ha永久樣區進行林分發展長期監測，並於樣區設置完成後隨即進行初次調查(Lin and Yen 2020)，樣區的設置方式係參考行政院農業委員會林務局

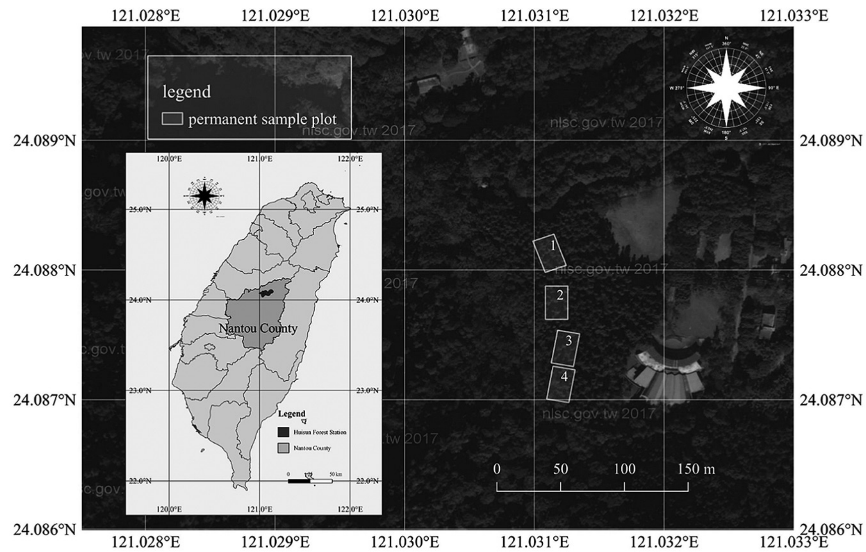

Fig. 1. Location of the study region and permanent sample plots (NLSC 2020).

所發行之森林永久樣區調查工作手冊規範(TFB 2006)，樣區設置規格採用17.6×28.4 m，本研究於2018年10月進行樣區複查，並詳細量測樣區內健全林木之DBH與H，以胸徑尺(YAMAYO diameter tape)測量DBH，及測高儀(Haga altimeter)與雷射測距儀(Leica DISTO X4)測量H，作為本研究之基礎資料。

二、研究方法

(一)樹高曲線式之建立

本研究整理過去文獻選出52種常見H-D model進行建模，有關H-D model之代號、型態及引用來源，根據線性及非線性型態彙整如Tables 1與2。本研究共採用12種線性型H-D model (Table 1)，如比照參數數目可分為2、3及4參數模式各有4種，且皆具備1個參數(參數a)作為常數項。另外在非線性型H-D model共採用40種(Table 2)，比照2、3及4參數模式分別為19、18及3種，且皆不具常數項。對於H-D model在DBH = 0時，H是否應通過1.3為過去許多學者在建模所討論之重點，Trorey (1932)及Meyer (1936)曾強調H-D model通過原點的重要性。然而Curtis (1967)則認為此原則並無絕對的必要性。本研究依據上述2種觀點，建立52種H-D model基本式，並比照基本式建立52種約束式。有關約束模式之操作需考量模式有無常數項之情形(Fig. 2)，在12種具備常數項之線性模式，係將a參數取代為1.3，而在40種無常數項之非線性模式中，除NLM35係以係數(h_0)調節模式讓其強制通過原點外，其餘39種模式係增加1.3於模式常數項，並將約束式之編號對照原式編碼尾端加上C作為區分(即模式編號-C)。本研究採用最小平方法(least squares method, LSM)求解所有H-D model之參數，以進行後續評估。

(二)樹高曲線式結構與模擬效果

1.約束效果之分析

為探討約束模式通過原點(DBH = 0, H = 1.3)之效果，本研究採用成對樣本t檢定(paired sample t-test)，檢測原式與約束式之差異，並參照Fig. 2分為線性及非線性2部分探討。由於在後續分析中，須由原式及約束式擇一進行，因此在2種模式型態中，當約束效果能顯著提升模擬能力，則採用約束形式進行後續分析；反

Table 1. Linear height-diameter models used in this study

No.	Equation form[1,2]	Reference	Number of parameters
LM1	$H = a + bD$	Larsen and Hann (1985)	2
LM2	$H = a + b\frac{1}{D}$	Arabatzis and Burkhart (1992)	2
LM3	$H = a + b\frac{1}{D^2}$	Curtis (1967)	2
LM4	$H = a + b \log D$	Curtis (1967)	2
LM5	$H = a + b\frac{D}{(D+1)} + cD$	Watts and Tolland (2005)	3
LM6	$H = a + bD + cD^2$	Trorey (1932)	3
LM7	$H = a + bD^{-1} + cD^2$	Curtis (1967)	3
LM8	$H = a + b \log D + c (\log D)^2$	Iman et al. (2017)	3
LM9	$H = a + bD + cD^2 + dD^3$	Pearl and Reed (1920)	4
LM10	$H = a + bD + cD^2 + d \log D$	Pearl and Reed (1920)	4
LM11	$H = a + bD^{-1/2} + cD^{-1} + dD^{-2}$	Curtis (1967)	4
LM12	$H = a + bD + cD^{-1/2} + dD^{1/2}$	Curtis (1967)	4

[1]) Constrained height-diameter model defined as a = 1.3.

[2]) H, height; D, diameter of breast height; a, b, c, and d, parameters.

Table 2. Nonlinear height-diameter models used in this study

No.	Equation form[1,2)]	Reference	Number of parameters
NLM1	$H = aD^b$	Stoffels and van Soest (1953)	2
NLM2	$H = \left(\frac{D}{a + bD}\right)^2$	Clutter et al. (1983)	2
NLM3	$H = \frac{D}{(a + bD)}$	Prodan (1965)	2
NLM4	$H = \frac{aD}{b + D}$	Menten and Michaelis (1913)	2
NLM5	$H = \frac{1}{a + bD^{-1}}$	Vanclay (1995)	2
NLM6	$H = a(\frac{D}{D + b})^2$	Osman et al. (2013a)	2
NLM7	$H = a\frac{D}{(1 + D)^b}$	Curtis (1967)	2
NLM8	$H = a\frac{1}{(aD^{-1})^b}$	Ogana (2018)	2
NLM9	$H = \frac{aD}{D + 1 + bD}$	Bates and Watts (1980)	2
NLM10	$H = a\left(\frac{D}{(1 + D)}\right)^b$	Curtis (1967)	2
NLM11	$H = 10^a D^b$	Larson (1986)	2
NLM12	$H = e^{(a + \frac{b}{D})}$	Osman et al. (2013a)	2
NLM13	$H = e^{(a + \frac{b}{D + 1})}$	Wykoff et al. (1982)	2
NLM14	$H = e^{a + b\,log\,(D)}$	Clutter et al. (1983)	2
NLM15	$H = a(1 - e^{-bD})$	Meyer (1940)	2
NLM16	$H = ae^{\frac{b}{D}}$	Schumacher (1939)	2
NLM17	$H = e^a D^b$	Huang et al. (2000)	2
NLM18	$H = aDe^{-bD}$	Huang et al. (2000)	2
NLM19	$H = a(ln(1 + D))^b$	Osman et al. (2013a)	2
NLM20	$H = \frac{D^2}{a + bD + cD^2}$	Strand (1959)	3
NLM21	$H = \frac{D^a}{b + cD^a}$	Osman et al. (2013a)	3
NLM22	$H = aD^{bD^{-c}}$	Sibbesen (1981)	3
NLM23	$H = aD^b e^{-cD}$	Fast and Ducey (2011)	3
NLM24	$H = aD^b e^{-cD^2}$	Fast and Ducey (2011)	3
NLM25	$H = \frac{a}{(1 + b^{-1}D^{-c})}$	Peschel (1938)	3
NLM26	$H = \frac{a}{(1 + be^{-cD})}$	Pearl and Reed (1920)	3
NLM27	$H = e^{(a + \frac{b}{D + c})}$	Ratkowsky (1990)	3
NLM28	$H = e^{a + bD^c}$	Curtis et al. (1981)	3
NLM29	$H = ae^{-be^{-cD}}$	Gomperz (1825)	3
NLM30	$H = ae^{\frac{b}{(D + c)}}$	Ratkowsky (1990)	3
NLM31	$H = ae^{(-bD^{-c})}$	Lundqvist (1957)	3

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NLM32	$H = a(1 - e^{-bD})^c$	Richards (1959)	3
NLM33	$H = a(1 - be^{-cD})$	Fang and Bailey (1998)	3
NLM34	$H = a(1 - e^{-bD^c})$	Weibull (1951)	3
NLM35[3]	$H = \left[h_0^b + (c^b - h_0^b) \times (\frac{1 - e^{-a \times (D - 0)}}{1 - e^{-a \times (100)}})\right]^{\frac{1}{b}}$	Schnute (1981)	3
NLM36	$H = (a + \frac{b}{D})^{-c}$	Osman et al. (2013a)	3
NLM37	$H = \frac{a}{b + e^{-cD}}$	Pearl and Reed (1920)	3
NLM38	$H = a(1 - be^{-cD^d})$	Bailey (1979)	4
NLM39	$H = a(1 - be^{-cD})^d$	Richard (1959)	4
NLM40	$H = a(1 - e^{-bD^c})^d$	Huang et al. (2000)	4

[1] Constrained height-diameter model defined by $H = 1.3 + f(D)$ except for NLM35.

[2] H, height; D, diameter of breast height; a, b, c, and d, parameters.

[3] The original form was defined by $h_0 = 0$, the constrained form was defined by $h_0 = 1.3$.

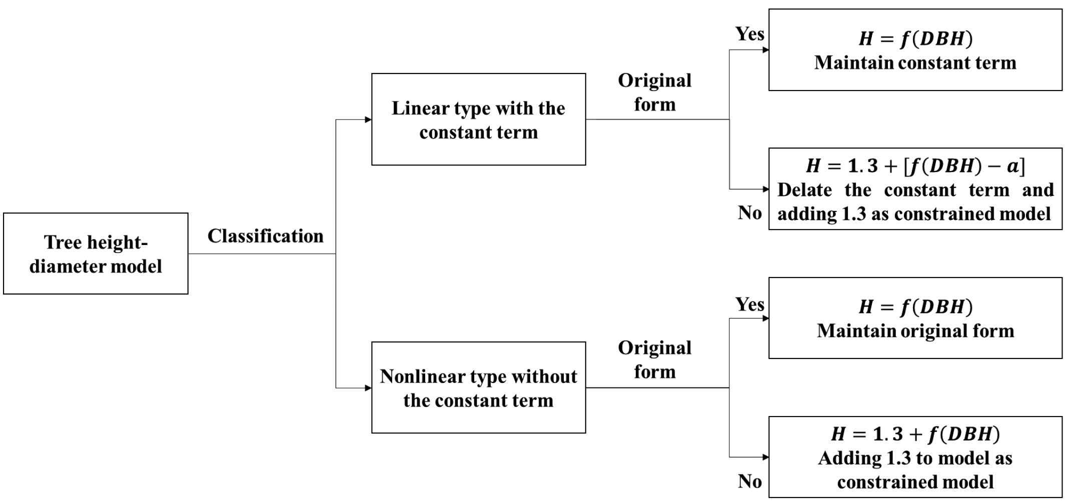

Fig. 2. Illustration of the original and constrained forms of the linear and nonlinear models.

之，約束效果無提升模擬能力之效果，則採用原式進行後續分析。

2.模式結構之分析

為探討參數數目及模式型態對模擬H之影響，本研究採用二因子變異數分析(two-way analysis of variance (ANOVA))，探討2、3及4參數因子(3種參數數目)及線性與非線性因子(2種模式型態)模擬H之差異。須注意的是在進行two-way ANOVA時，若交互項(交感作用)呈現顯著，則需再採用限制的方式進行ANOVA，進一步分析主因子之效果，即表示在分析參數數目的差異，須再分為線性及非線性2種型態各別進行；而在分析模式型態的差異，須再分為3種參數數目各別討論。而在事後檢定(post hoc)，本研究採用Bonferroni test比較不同因子組合之差異。為避免連續5次(3種參數數目+2種模式型態)ANOVA產生型I錯誤率膨脹，此部分採用族系錯誤率，將檢定α值(0.05)設定為0.05/5=0.01，使整體的型I錯誤率維持在0.05的水準。

(三)模式評估準則

1.模式評估指標

本研究採用3種評估指標探討模擬結果之良窳，分別為誤差平方和(residual sum of squares (RSS))、誤差均方根(root mean square error (RMSE))及修正的赤池信息準則(Akaike information criterion (AIC))，各評估指標之計算公式如Table 3所示。上述3種指標值越小，則表示模擬結果較佳。

2.模式優劣排序

為比較不同H-D model模擬效果之優劣順序，本研究採用相對排名法(relative rank of method)，加總模式在各指標值所得之相對排名分數(relative rank (R-rank))，作為排名模式之依據，其公式如(1)式所示。

$$R_i = 1 + \frac{(m - 1)(S_i - S_{min})}{S_{max} - S_{min}} \quad (1)$$

(1)式中，R_i為第i個(i = 1, 2, ⋯, m) H-D model所得之R-rank；m為模式之總數量；S_i為第i個H-D model所得之檢定值；S_{max}為S_i之最大值；S_{min}為S_i之最小值。當R_i值越小，表示模式表現較優良，此方法不僅可排序模式優劣順序，亦可用以檢視模擬結果之差距(Poudel and Cao 2013)。

結 果

一、樹高曲線式之建立

本研究調查4個永久樣區共計104株同時具備DBH及H之樣木，所得結果顯示，單木DBH平均值為36.28±7.28 cm，DBH介於19.40至55.05 cm之間，H平均值為21.81±2.93 m，H介於14.10至27.75 m之間。經最小平方法推導H-D model所建立52種基本原式及52種約束式，茲將模式分為線性與非線性模式討論，所得參數、RSS、RMSE、AIC及R-rank之敘述性統計結果整理如Table 4所示。由於每種模式特性不同，因此各參數具有很大的標準差。比較評估指數之結果，不論在原式或約束式，非線性模式皆小於線性模式，即表示非線性模式表現較佳。另外，比對原式與約束式的表現可知，線性模式受約束後，4種評估指標之平均值皆上升，而非線性模式經約束後，評估指標之平均值則些微降低。

比較104種H-D model在R-rank之表現，所得前10名模式之參數與評估指標如Table 5所示，由評估指標可知，10種模式在整體表現差異不大，特別在排序第6至第10名模式，評估指數差距皆小於0.01。藉由10種模式比較模式型態(線性與非線性)之表現可知，第1至第5名皆為線性模式，而第6至第10名則皆為非線性模式，有關各模式模擬表現整合如Fig. 3所示，由於非線性模式之間表現相近，因此以最佳模式NLM11-C作為表示，由Fig. 3可知，模式之間的差異主要在觀測值DBH兩端極值(DBH > 50 cm及DBH < 25cm)的模擬，值得注意的是，LM11及LM11-C出現曲線斜率為負值之情形，此結果為模式過於緊密匹配特定樣本，產生有別於常理之現象，為模式之過度配適(overfitting)。

Table 3. Criteria for model performance in this study

Performance criterion	Function[1)]
Residual sum of squares (RSS)	$\sum_{i=1}^{n}(y_i - \hat{y}_i)^2$
Root mean square error (RMSE)	$\sqrt{(RSS/(n - p))}$
Akaike information criterion (AIC)	$2p + n\,ln\frac{RSS}{n} + \frac{2p\,(p + 1)}{n - p - 1}$

1) In the function, y_i is observations, $\hat{y}_i$ is predictions, n is number of sample trees, and p is number of parameters of the model.

Table 4. Results of coefficients, residual sum of squares (RSS), root mean square error (RMSE), Akaike information criterion (AIC), and relative rank (R-rank) for the various height-diameter models

Item	Linear model (n = 12)		Nonlinear model (n = 40)	
	Original	Constrained	Original	Constrained
a	-18.70	-	8567.79	9192.19
	±182.79[1)]	-	±53362.08	±58075.27
b	-363.00	1700.92	18.22	80.70
	±1364.00	±5367.12	±422.97	±767.20
c	-400.69	-481.72	4.19	3.96
	±1668.99	±1385.12	±18.87	±17.85
d	7612.78	7000.66	1.41	1.22
	±15263.25	±13995.14	±8.34	±7.11
RSS	449.66	1805.12	447.22	446.67
	±29.58	±3324.00	±7.36	±6.78
RMSE	2.11	3.32	2.10	2.10
	±0.06	±2.68	±0.01	±0.01
AIC	158.39	214.32	157.08	156.95
	±5.43	±112.72	±1.27	±1.20
R-rank	5.67	49.88	5.12	5.06
	±2.65	±96.91	±0.59	±0.55

[1)] mean±standard deviation.

Table 5. Evaluation statistics of coefficients, residual sum of squares (RSS), root mean square error (RMSE), Akaike information criterion (AIC) and relative rank (R-rank) of the top ten models

ID	Parameter				Criterion			
	a	*b*	*c*	*d*	RSS	RMSE	AIC	R-rank
LM11	-6.54	762.81	-4406.06	30,507.11	424.21	2.05	152.77	3.00
LM11-C	-	644.57	-3909.46	27,993.37	424.85	2.05	152.77	3.01
LM10	148.85	4.20	-0.03	-156.24	428.28	2.07	155.60	4.12
LM1	11.48	0.28	-	-	442.30	2.08	154.67	4.13
LM5-C	-	10.78	0.28	-	442.49	2.08	154.83	4.18
NLM11-C	0.54	0.50	-	-	444.09	2.09	155.09	4.32
NLM14-C	1.23	1.15	-	-	444.09	2.09	155.09	4.32
NLM1-C	3.44	0.50	-	-	444.09	2.09	155.09	4.32
NLM17-C	1.23	0.50	-	-	444.09	2.09	155.09	4.32
NLM8-C	11.74	0.50	-	-	444.09	2.09	155.09	4.32

二、約束效果對樹高曲線式之影響

經成對樣本t-test分析約束模式通過原點之效果，所得線性與非線性模式在3種評估指標之結果如Table 6所示。結果顯示，約束效果僅在非線性模式呈顯著，即表示約束模式通過原點能降低非線性模式之評估指標值，然而對照非線性模式所得之差異平均值($\bar{d}$)可知，此效果所增加非線性模式的效益幅度不大。另外，在線

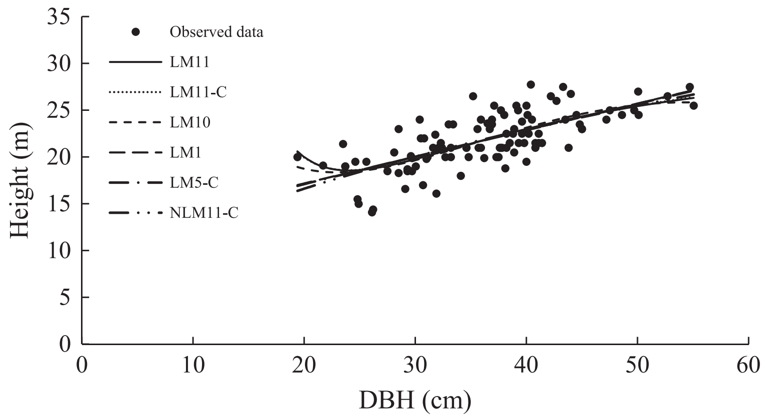

Fig. 3. Distribution observations and predicted curves for the relationship between tree height and diameter at breast height for Taiwania.

Table 6. Paired sample *t*-test for residual sum of squares (RSS), root mean square error (RMSE), and Akaike information criterion (AIC) between the original models and constrained models for linear and nonlinear height-diameter models

Criterion	Linear (n = 12)		Nonlinear (n = 40)	
	$\bar{d}$[1)]	t-value	$\bar{d}$	t-value
RSS	1355.462±3295.601	-1.425	-0.553±0.786	4.483[***2)]
RMSE	2.619±1.205	-1.595	-0.001±0.002	4.471[***]
AIC	55.927±107.807	-1.797	-0.127±0.179	4.455[***]

[1)] $\bar{d}$ is the mean difference between the original and constrained models.

[2)] [***] Indicates a significant differences at $p < 0.001$ by the paired sample t-test.

[3)] Values are the mean±standard deviation.

性模式則皆無顯著效果，因此在後續分析中，在線性模式將採用原式，而在非線性模式則採用約束式。

三、參數數目對樹高曲線式之影響

為探討模式型態與參數數目對H-D model的影響，將模式所得RSS、RMSE及AIC進行two-way ANOVA，所得結果整合如Table 7所示。各評估指標皆在交互項呈現顯著，表示模式型態與參數數目具交互作用(交感作用)，必需再限制主因子進行ANOVA，以排除交互作用對分析的影響。

經限制因子進行ANOVA，所得3種評估指標之結果彙整如Table 8所示，在參數數目的結果顯示，僅線性模式呈顯著，經事後檢定顯示(Fig. 4A)，為2參數顯著大於3及4參數，且3及4參數並無顯著差異，除在AIC之結果中，2及3參數無顯著差異，與RSS及RMSE有所區別。而模式型態的結果顯示，僅在2參數模式呈現顯著，經事後檢定顯示(Fig. 4B)，主要為線性型態大於非線性型態。

討 論

一、樹高曲線式之表現

一般而言，DBH和H具有顯著的正向相關性，然而這種關聯性會隨著樹種和不同的生長過程階段而改變(O'Brien et al. 1995, Chen et al. 1996, Tsogt et al. 2013, Chiu et al. 2017)，因此在建立H-D model必須考慮樹種在不同林木生長

Table 7. Two-way analysis of variance (ANOVA) for residual sum of squares (RSS), root mean square error (RMSE), and Akaike information criterion (AIC) with model type and the number of parameters

Main factor and interaction term	Criterion					
	RSS		RMSE		AIC	
	F-value	*p*-value	*F*-value	*p*-value	*F*-value	*p*-value
Model type (M-t)	1.470	0.231	1.846	0.181	1.290	0.262
Number of parameters (N-p)	19.383	0.000[1)]	9.475	0.000[1)]	3.696	0.032
M-t×N-p	6.985	0.002	6.056	0.005	6.580	0.003

[1)] $p < 0.001$ by a two-way ANOVA.

Table 8. *F*-values for the analysis of variance (ANOVA) for residual sum of squares (RSS), root mean square error (RMSE), and Akaike information criterion (AIC) for each combination

Criteria	Factor				
	Number of parameters		Model type		
	Linear (n = 12)	Nonlinear (n = 40)	2 (n = 23)	3 (n = 22)	4 (n = 7)
	(2~4 parameters)		(2 different model types)		
RSS	19.146[*1)]	3.422	16.453[*]	0.050	1.822
RMSE	11.301[*]	0.725	15.548[*]	0.052	1.129
AIC	7.121[*]	0.554	15.240[*]	0.055	1.843

[1)] [*] Indicates a significant difference at $p < 0.01$ (family-wise error rate) by the ANOVA.

階段的特定時間來建立，以其用DBH推估林木之高度。尤其在地區性之H-D model的研究，常需探討不同生長階段H-D model之建立。本研究所探討之對象為壯齡期之臺灣杉人工林，在整體模式的表現顯示非線性模式較佳(Table 4)，與多數研究所得結果相符(Huang et al. 2000, Feldpausch et al. 2011, Mehtätalo et al. 2015, Sharma et al. 2016)。然而在個別模式表現結果顯示，最佳5名模式皆屬線性模式(Table 5)，此現象與Liu et al. (2017)建立水杉(*Metasequoia glyptostroboides* H.H. Hu & W.C. Cheng) H-D model的研究相似。有關H-D model之應用，一般多直接採用評比模式的相關指標篩選表現較優秀的模式，然而各指標評比基準有所不同，在採用多種指標評比的情形下，結果容易不一致，本研究考量模式表現之誤差量與模式結構之複雜度，分別選用RSS、RMSE及AIC三種指標，再參考Poudel and Cao (2013)的方式，將前3種指標整合為R-rank進行評估。所得結果顯示，LM11與LM11-C所模擬之結果最符合DBH與H之實際分布，值得注意的是，此2者模式在模擬DBH < 25 cm之區段出現負斜率之情形(Fig. 3)，有違一般常理上所認為DBH與H之正向關係，然而在人工林發展階段中，其關係可能會受到樹種在不同生長階段之特性、經營撫育方式及自然外力(如：風折)等因素，導致負相關之情形出現，過去在Yen et al. (2008)探討柳杉(*Cryptomeria japonica* (Thunb. ex L. f.) D. Don)人工林的研究中，亦曾發現DBH與H之關係曲線斜率出現負值之現象。然而H-D model出現負斜率之狀況，容易在後續的應用產生較大的誤差(Meyer 1936)，就此觀點而言，本研究將LM11及LM11-C排除後續模式選拔，因此整體排序依序調整，依R-rank排名順序最佳模式為LM10，其次為LM1 (Table 5)。然本研究所建立之H-D model屬40年生人工林之模式，株數

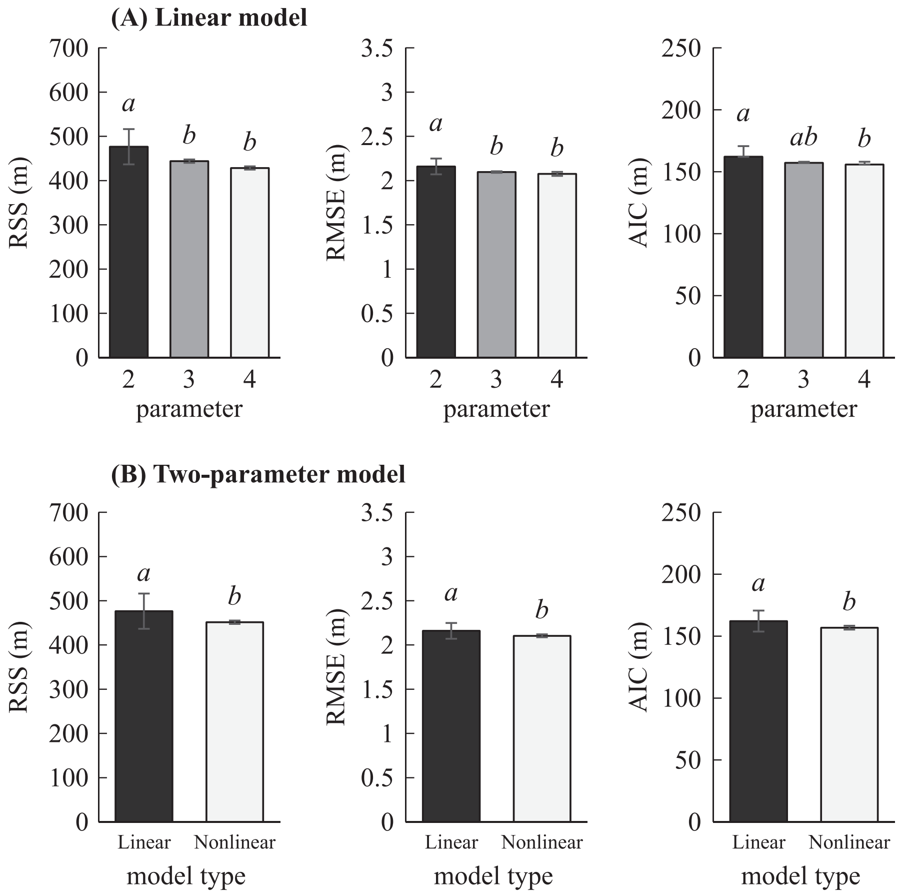

Fig. 4. Comparisons of the sum of squares (RSS), root mean square error (RMSE), and Akaike information criterion (AIC) among different numbers of parameters and model types by the Bonferroni test at *p* < 0.01 (family-wise error rate). Different letters denote a significant difference according to the Bonferroni test.

密度為520±58.88 trees ha^{-1}，且優勢木樹高為25.63±2.17 m，在應用上仍須注意相關林分屬性(如：林齡、地位、林分密度)，即使用上的限制。

二、樹高曲線式結構之分析

許多研究在建立H-D model會將模式於DBH接近0 cm時，設定H收斂在1.3 m，此觀點為Meyer (1936)提出並被許多研究所應用，然而也有研究主張，約束模式通過原點主要在於改善小徑木之模擬，對於只探討大徑木之應用並非必然(Curtis 1967)，亦有探討H-D model應用於不同林分結構的研究顯示，在部分約束模式通過原點的結果中，模式彈性會受到限制而導致配適能力下降(Newton and Amponsah 2007)。而本研究所採用之人工林屬中大型徑木，在結果顯示約束處理可提升非線性模式之模擬效果，然效果有限，各指標提升解釋比例依序為RSS: 0.12%; RMSE: 0.06%; AIC: 0.08%。值得注意的是在線性模式的結果雖不顯著，卻呈現下降的趨勢(Table 6)，推測其不顯著之原因為各模式變化幅度差異較大所導致。

在多數研究顯示，H-D model模擬效果會隨參數數目的增加而提升(Abedi and Abedi

2020, Bayat et al. 2020, Lebedev and Kuzmichev 2020)，然而本研究僅在線性型態2參數與3參數模式之間有此現象，唯當3參數增至4參數時，其效果並不顯著，此部分之比較在過去研究多為直接採用評估指標進行說明，而本研究將模式表現加以two-way ANOVA進行探討。另外在模式的模擬上，模擬能力隨著參數數目增加的情形並非皆然，參數的增加雖可能獲得較好的模擬結果，卻容易導致模式無法收斂，模擬的成功率也會隨之降低(Fang and Bailey 1998, Osman et al. 2013b)。此情形與本研究模式在3參數的模擬成功率(95%)低於2參數之情形相若。

有關模式在線性與非線性之模擬，除了直接以評估指標得知非線性表現較佳外，本研究再採用two-way ANOVA得知此差異在2參數最為明顯，而在3與4參數並不顯著。綜合在參數數目與模式型態之結果，2參數線性模式之類型雖在整體表現較不佳，然而其類型屬最簡單推估的方式(Mehtätalo et al. 2015)，在個別模式表現中仍有為佳者(LM1)，因此仍有測試此類型模式之效益(Scaranello et al. 2012)。在另一方面，2參數線性模式模擬H的方式，近似於DBH與H的之相關分析，因此亦可作為2者性狀值相關性之參考。

結論

本研究為探討不同H-D model之結構在模擬臺灣杉人工林之效果，所得之結論與建議如下：

一、經綜合評估結果顯示，LM10為最佳模式，亦為最佳線性模式，其次為LM1。而NLM11-C為最佳非線性模式，其次為NLM1-C、NLM8-C、NLM14-C及NLM17-C，此5種模式表現差異不大。

二、約束H-D model通過原點可提升非線性模式之模擬能力，然而在線性模式雖無顯著效果，卻出現模擬能力下降之趨勢。綜合上述，本研究建議在非線性模式採用約束式，而在線性模式採用基本原式。

三、影響H-D model模擬之因子中，在參數數目影響方面，結果僅在線性模式中2與3參數模式之間，參數數目的增加可顯著提升模式模擬之表現；而在模式型態的影響方面，結果僅在2參數模式中，非線性模式表現顯著較佳。

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