Research paper

Taper Modeling on Taiwania Plantation Trees in the Liukuei Area

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【Summary】

Taper is used to measure the rate of decrease in the stem diameter of a tree from the bottom upwardly. Taper equations express the expected stem diameter with or without bark, as a function of the height above ground level, total tree height, and the diameter at breast height. Five tapering modeling approaches were used to estimate the stem diameter at a given height above the stump in Taiwania plantations, and their levels of accuracy and precision were compared. The results indicated that based on 3 criteria simultaneously considered, the trigonometric taper modeling approach was the worst for describing the taper of the entire tree bole, followed by the sigmoid form approach. Three-segmented polynomials, the variable-form stem profile and the polynomial form with higher-order approaches were preferred to describe the taper of the entire stem. The mean relative biases in percentage for these models on the validation trees were all < 4%. Along the bole, the root swell (the segment from 0.3 to 1.3 m in height) was the most difficult part to predict by taper modeling. However, the precision and accuracy of the prediction of tree root swell can be significantly improved using the 3-segmented polynomials or variable-form taper models.

Key words: stem profile, variable form model, segmented polynomials model.

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研究報告

六龜地區台灣杉人工林林木尖削度模式之建立

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摘　要

林木尖削度(taper)是指樹幹不同部位直徑之尖削程度，不同形狀之樹幹其尖削程度不一。尖削度模式又稱為樹幹剖面模式(stem profile)，是描述樹幹連皮或去皮直徑隨著某特定高度之變化情形。尖削度之重要性是在可以計算一株樹在不同利用標準下之木材利用材積。因此，對木材工業而言，良好之尖削度模式可以在不同標準規格下提供準確有效之木材利用率。一般言之，尖削度模式種類甚多其

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配置之方式亦不同。本研究使用五種不同類別之模式探討六龜地區台灣杉人工林林木尖削度模式之建立，並就各模式之推估能力進行比較。研究結果顯示就總體之推估能力(含偏差與精密度)言之除三角模式外，各模式之間之差異不大，但描述各不同段位之能力則有明顯之差別。以從根株至10%樹高高度之部位觀之，幾何形狀一般式和單一低階多項式之預測能力顯然較變動形數模式和分段式多項式為差。變動形數模式和三段式多項式，雖然其數學模式較為複雜，然不但其總體之偏差百分比小於4%，其幹材較低部位之偏差亦為最低，因此，變動形數模式和三段式多項式是為比較理想之尖削度模式。

關鍵詞：樹幹剖面模式、變動形數模式、分段式多項式。

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INTRODUCTION

The subject of stem taper curves constitutes 1 of the bases of mensuration and biometrics (Husch et al. 1972). Accurate estimates of timber volume and dimensions are 2 of the most critical pieces of information in the timber industry. As a result of closer utilization of wood in the stem and of more-intensive forest management, the need has arisen for more-accurate taper functions that describe the profile of the stem along its entirety (Reed and Green 1984).

In general, the stem form of conifers can be described as a combination of (*i*) within the live crown, where the stem is usually conical in form; (*ii*) in the region of the root swell near the base of tree, where the stem is neiloidal; and *(iii*) over the main section of the stem between the root swell and the base of the crown, where the stem is paraboloidal (Husch et al. 1972). While the form and taper of tree stems are 2 terms have been used interchangeably in the past, it is now generally accepted that form refers to the geometric shape of the stem, whereas taper refers to the rate of decrease in diameter with the increase in height up the stem (Newnham 1992). Moreover, taper is also called as the stem profile (Amidon 1984, Valentine and Gregoire 2001).

Modeling the stem taper has been a widespread effort in forestry during the past century. Since the early 19th century, researchers have worked out methods to express tree form and taper in terms of easily measured tree characteristics (Perez et al. 1990). Early efforts to develop taper functions to describe the upper stem diameter in relation to height focused on the merchantable portions of the stem and were relatively simple in formulation and thus did not satisfactorily describe the root swell and tip (Bruce et al. 1968, Newnham 1992). Because of the geometric flexible nature of tree stems, many different models of varying complexity were proposed during the past several decades in attempts to describe tree taper more accurately (Max and Burkhart 1976).

A literature review showed that the approaches adopted for modeling taper during the past can be generally classified into the following categories: (*i*) sigmoid form (Ormerod 1973, Biging 1984); (*ii*) polynomials of order 2 or greater (Bruce et al. 1968, Kozak et al. 1969); (*iii*) variable-form or variable-exponent stem profile models (Newberry and Burkhart 1986, Newnham 1992, Kozak 1988, Bi 2000); (*iv*) segmented polynomials with submodels (Max and Burkhart 1976, Cao et al. 1980); (*v*) trigonometric taper models (Thomas and Parresol 1991); (*vi*) compatible taper models (Byrne and Reed 1986, Rustagi and Loveless 1991, Zhang et al. 2002, Jordan et al. 2005); and (*vii*) a nonparametric technique (M'Hirit and Postaire 1985). The polynomial approaches treat the stem as an entire bole. Therefore, a single polynomial function is used with 2 or more terms in the regression equations, and terms are raised to powers as high as 40 to ensure a good fit at the base of the stem (Bruce et al. 1968). Advocates of the variable form approach consider variations in form between and within trees, and developed functions based on the assumption that within a tree the form of the stem (i.e., the geometric shape) varies continuously along the stem (Newberry and Burkhart 1986, Kozak 1988, Perez et al. 1990, Newnham 1992).

Taiwania (*Taiwania cryptomerioides*) is the major plantation species in the Liukuei Experimental Forest. The inventory shows that by the end of 1991, 1560 ha of forest was in plantations, accounting for approximately 16.22% of the total area. Among these, 51.6% were covered by Taiwania (TFRI 1992). While several studies regarding Taiwania plantation growth have been conducted in the past (Chen et al. 1997, Chen and Huang 1999, Wang et al. 2004), no work on stem profiles of Taiwania plantation trees has been done. Therefore, the purpose of this study was to develop a taper model and to compare the accuracy and precision of taper prediction among several approaches.

MATERIALS AND METHODS

Data for this study came from Taiwanian plantations of Liukuei Experimental Forest of Taiwan Forestry Research Institute (TFRI), southwestern Taiwan. Sample trees from plantations of Taiwanian in compartment 3, 10, 12, 14, 18, 20, and 24 were included in this study. Single-stemmed trees without broken tops of a variety of tree sizes in diameter at breast height (DBH) and in total height were selected for felling. Diameters on the outside of the bark were measured at ground stump height (0.3 m), breast height (1.3 m) and every 1-m height interval above breast height. The total height and crown base height were also recorded for each individual tree. Moreover, the boles of trees were cut into sections. The discs at stump height, breast height, and 2-m intervals in height above 1.3 m for each tree were carried into the laboratory for a stem analysis. All individual measurements of diameter and height pairs for trees were randomly divided into fitting and validation subsets. Seventy percent of the sample, or 51 trees with 1208 diameter/height observations, was used for fitting the taper models, and the remaining 22 trees with 461 observations were used for model testing.

Five tapering modeling approaches were applied and compared in this study. The associated mathematical expressions of formulation for each was given below

1. Sigmoid form approach

The Chapman-Richards function is widely used to estimate tree growth patterns. Biging (1984) claimed that the sigmoid form usually shown in tree growth can be also used to describe the tree stem profile through an effective transformation. Therefore, based on the integral form of the Chapman-Richards function, Biging proposed a taper model for second-growth mixed conifers in northern California. Equation (1) shows the formula of this model:

$$d = DBH*\{b_1 + b_2* \ln [1 - \lambda (h / HT)^{1/3}]\}; \quad (1)$$

where d is the diameter outside and/or the diameter inside bark (cm) at a specific height h from the ground, DBH is the diameter outside bark at breast height (cm), h is the height at a specific point on the bole (m) from the ground, HT is the total tree height (m), λ is $1- \exp(-b_1 / b_2)$, and b_1 and b_2 are estimated parameters.

Equation (1) implies a constrained form of the taper equation that is forced to go through the tip of the tree (i.e., $d = 0$ when $h = HT$).

In addition to the growth function used to express the tapering sigmoid form, Ormerod (1973) proposed a simple-form taper model as shown as equation (2):

$$d = DBH*[(HT - h) / (HT - 1.3)]^{b1}; \quad (2)$$

where b1 is an estimated parameter.

All other variables in equation (2) are the same as those in equation (1). This function has been so conditioned that d equals to 0 at the top of tree and d equals DBH at the height of 1.3 m. Moreover, if the fitted exponent b1 is < 1, the shape of the hole will be parabolic, and if > 1, then it will be neiloidal.

2. Polynomial form approach

A. Lower order

Kozak et al. (1969) proposed a polynomial taper model with a degree of 2:

$$(d / DBH) = b_1*(h / HT - 1) + b_2*[(h / HT)^2 - 1]; \quad (3)$$

All variables in equation (3) are the same as those in equation (1). The same as equation (1), equation (3) satisfies the constraint of the tip pass requirement.

B. Higher order

To improve the accuracy and precision in describing the root swell and tip, Bruce et al. (1968), using the relative height rather than the absolute height, derived a polynomial taper model with high orders up to 40:

$$d^2 = DBH^2*\{ b_1*X^{1.5}*(10^{-1}) + b_2*(X^{1.5}- X^3)*DBH*(10^{-2}) + b_3*(X^{1.5}- X^3)*HT*(10^{-3}) + b_4*(X^{1.5}- X^{32})*HT*DBH*(10^{-5}) + b_5*(X^{1.5}- X^{32})*HT^{1/2}*(10^{-3}) + b_6*(X^{1.5}- X^{40})*HT^2*(10^{-6})\}; \quad (4)$$

where X is $(HT - h) / (HT - 1.3)$, and all other variables are the same as those in equation (1). Equation (4) also meets the tip pass requirement.

3. Segmented polynomials with submodels

Segmented polynomial models consist of a sequence of grafted submodels. In the case of the taper profile, the entire bole is partitioned, and a different polynomial submodel is defined for each section of the partition. Then, these submodels are grafted to form the segmented polynomial model. Generally, to meet the smooth requirement of the stem profile, restrictions that function for each submodel must be continuous and have continuous first- or higher-order derivatives have to be imposed on the model (Gallant and Fuller 1973).

In the case of 1 joint point of 2 submodels, Max and Burkhart (1976) proposed a 2-quadratic segmented submodel as Equation (5):

$$d^2 / DBH^2 = b_1*(h / HT - 1) + b_2*[(h / HT)^2 - 1] + b_3*(\alpha - h / HT)^2*I_+ (\alpha - h / HT); \quad (5)$$

where d is the diameter outside and/or the diameter inside bark (cm) at a specific height h from the ground, DBH is the diameter outside bark at breast height (cm), h is the height at a specific point on the bole (m) from the ground, HT is the total tree height (m), α is a joint point of the submodels, $I_+ (\alpha - h / HT)$ is a dummy variable with a value of $I_+ (\alpha - h / HT)$ either = 1, if $\alpha \geqq h / HT$, or = 0, otherwise, and b_1, b_2, b_3, and α are estimated parameters.

Equation (5) can be extended to the case of 2 joint points of 3 submodels as in Equation (6):

$$d^2 / DBH^2 = b_1*(h / HT - 1) + b_2*[(h / HT)^2 - 1] + b_3*[(\alpha_1 - h / HT)^2*I_+ (\alpha_1 - h / HT)] + b_4*[(\alpha_2 - h / HT)^2*I_+ (\alpha_2 - h / HT)]; \quad (6)$$

where α_1 and α_2 are 2 joint points of the submodels, $I_+ (\alpha_1 - h / HT)$ is a dummy variable with a value of $I_+ (\alpha_1 - h / HT)$ either = 1, if $\alpha_1 \geqq h / HT$, or = 0, otherwise, $I_+ (\alpha_2 - h / HT)$ is a dummy variable with a value of $I_+ (\alpha_2 - h / HT)$ either = 1, if $\alpha_2 \geqq h / HT$, or = 0, otherwise, and b_1, b_2, b_3, b_4, α_1, α_2 are estimated parameters.

Parameters b_1 and b_2 in equation (6) are used to describe the profile of the tree top section with a condition of tip pass requirement. Parameters b_3 and b_4 are designed to represent the middle and lower sections of the tree bole, respectively. Equation (6) is also referred to as a quadratic-quadratic-quadratic model.

4. Variable-form stem profile models

While the advantage of the segmented submodels approach is that diameters are predicted with less bias than by a single function representing most parts of stem, the associated disadvantage is the difficulty in estimating the parameters (Kozak 1988). An alternative approach, the so-called a variable-form taper function, was introduced by Kozak (1988) and Newnham (1992) to describe the shape of the stem with a continuous function using a continuously changing exponent in a single function to compensate for the form changes of different tree sections.

The variable-exponent taper model proposed by Kozak (1988) has the following form:

$$d = b_1 * DBH^{b2} * b_3^{DBH} * Y^C; \quad (7)$$

where d is the diameter outside and/or the diameter inside bark (cm) at a specific height h from the ground, DBH is the diameter outside bark at breast height (cm), h is the height at a specific point on the bole (m) from the ground, HT is the total tree height (m), Y is (1 - sqrt (Z)) / (1 - sqrt (I)), Z is h / HT, I is the location of the inflection point, C is $b_4*Z^2 + b_5*\ln(Z + 0.001) + b_6*\text{sqrt}(Z) + b_7*e^Z + b_8*(DBH / HT)$, and $b_1 \sim b_8$ are parameters to be estimated.

Equation (7) has the property that the diameter equals 0 at the top of the tree. In addition, d equals the estimated diameter at the inflection point and the function changes the direction when h / HT = I.

5. Trigonometric taper models

In addition to the polynomials used to describe the taper curve, trigonometric functions have also been applied to model the bole taper due to the fact that trigonometric functions on the unit circle have a direct analogy to the relative height vs. relative diameter plots shown in many taper equations. Thomas and Parresol (1991) proposed a trigonometric taper function represented by the equation (8):

$$d^2 / DBH^2 = b_1*(R - 1) + b_2*\sin(c*\pi*R) + b_3*\cot(\pi*R/2); \quad (8)$$

where d is the diameter outside and/or the diameter inside bark (cm) at a specific height h from the ground, DBH is the diameter outside bark at breast height (cm), h is the height at a specific point on the bole (m) from the ground, HT is the total tree height (m), arguments for trigonometric functions are expressed in radians, R is h / HT, c is 1.5 for the softwood, and 2.0 for the hardwood, and b_1, b_2, and b_3 are estimated parameters.

The precision and accuracy of all tree profile prediction systems were evaluated by using the following 5 criteria in terms of diameters at different bole heights:

1) average bias (cm) = $\Sigma (d_i - d_{ihat}) / n$,

2) average bias (%) = $\Sigma ((d_i - d_{ihat}) / d_i*100) / n$,

3) average absolute bias (cm) = $\Sigma \text{abs}((d_i - d_{ihat}) / n)$,

4) standard error of the estimate (SEE) (cm) = $\text{Sqrt}(\Sigma (d_i - d_{ihat})^2 / (n - m - 1))$, and

5) Mean squared error (MSE) = $\text{Bias}^2 + \text{Variance}$;

where d_{ihat} is predicted at height h_i and d_i is the actual measurement at point *i* with height h_i on the bole, m is the number of parameters in the models, and n is the number of points in a specified region of relative height, say 0.1~0.2 of total height.

Since it reveals the potentially different capabilities inherent in describing tapering on the portion of the stem of the models used in this study, the comparisons of prediction power among models should focus not only on the entire stem but also on individual parts of the stem (Biging 1984, Bailey 1994). In this study, a relative height with 5 levels (e.g., 10~20%, 20~40%, 40~60%, 60~80%, and 80~100%) of the total height was used to identify segments of the stem above the DBH height.

RESULTS AND DISCUSSION

As smaller trees have limited potential for multiproduct utilization and huge trees are rare and often unavailable for commercial harvest, only trees of Taiwanina plantations with DBH ranging from 17.5 to 55.0 cm were used in this study. Bark thickness was determined from a stem analysis to estimate the diameter inside the bark. The distribution of DBH and tree height on sample trees is shown in Table 1. In this study, measurements at ground level were not used because they were suspected of degrading model performance in the lower main stem, and dimensions below the stump level are less important than the lower bole for commercial utilization standards (Czaplewski and McClure 1988).

Observations of sample trees used to estimate the parameters were randomly and proportionally selected from each 5 cm of diameter at breast height and 2-m height classes. Selection of observations in this way provided a wide range of tree sizes, which is necessary for good parameter estimation (Kozak 1988). Usually, when taper equations are fitted, the parameters are estimated from a subset of the data (fitting dataset), and then the equation is evaluated on the remainder of the observations (validation dataset). The estimated coefficients and their standard errors for all models are presented in Table 2.

The ratio of DBH to total height HT is an indicator that is widely used to estimate the form or the slenderness of tree boles (Hann et al. 1987, Kozak 1988). Figure 1 shows the effect of DBH/HT on taper based on observations of 2 sample trees. The taper curve on a higher slender tree (i.e., less DBH/HT) in the lower stem first lies below that on a less slen-

Table 1. Distribution of sample trees by diameter at breast height (DBH) and total height

DBH (cm)	Height (m)											
	< 10	10~12	12~14	14~16	16~18	18~20	20~22	22~24	24~26	26~28	28~30	> 30
< 10												
10~15												
15~20				3	3							
20~25			2	3	2	1						
25~30			1	1	2	3	7	1				
30~35					3	6	5	4				
35~40						6	5	2				
40~45						3	4	3		1		
45~50							1					
50~55												1
Total			3	7	10	19	22	10		1		1

Table 2. Estimated coefficients and their standard errors (in parentheses) of the diameter outside the bark for all models based on the fitting data

Model equation	Parameters								
	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	α_1	α_2
1	1.3024 (0.0064)	0.5514 (0.00881)								
2	0.8037 (0.0071)									
3	-0.8055 (0.0259)	-0.2849 (0.0206)								
4	9.93093 (0.0701)	-0.92173 (0.24949)	21.91413 (4.15771)	-15.93014 (1.75591)	17.01946 (2.47392)	-160.31739 (30.04769)				
5	-1.60041 (0.0337)	0.56437 (0.02586)	87.55962 (2.0045)						0.1	
6	-3.80657 (0.4759)	1.84400 (0.27653)	-1.49027 (0.32066)	91.26368 (2.13497)					0.1	0.8
7	0.9068 (0.1669)	0.9827 (0.0744)	0.9986 (0.00228)	1.6325 (0.1476)	-0.3710 (0.0342)	2.3590 (0.3448)	-1.3042 (0.1833)	0.18253 (0.0103)		
8	-0.99795 (0.0100)	-0.05134 (0.00836)	-0.000160 (0.000186)							

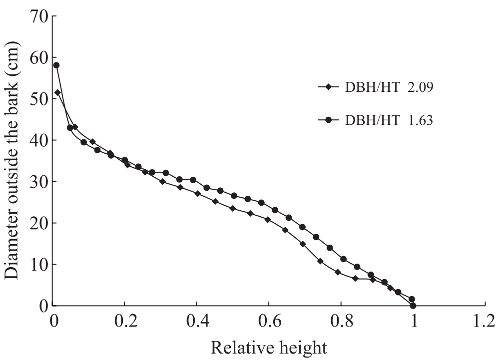

Fig. 1. Effects of diameter at breast height (DBH)/total heigh (HT) on taper based on observations of 2 sample trees.

der tree, then crosses at a point and reverses direction afterwards. In other word, given the same DBH, different DBH/HT trees will result in various tapering rates at different portions of stem. Consequently, it is desirable to take DBH/HT into account in fitting the taper models.

A positive correlation between the taper rate and height was demonstrated in this study for both the diameter outside the bark (r = 0.565, p = 0.000) and the diameter inside the bark (r = 0.497, p = 0.000). Generally, a remarkable taper rate (over 10% on average for the fitted trees) was found in the bole between the stumpage height and breast height. Then a less notable taper rate change was detected with an increase in bole height up to the height at the crown base (HCB). Finally, an eminent taper rate occurred again within the crown. This finding supports that taper rates below the crown are less than those within the crown (Larson 1963).

The diameter inside the bark at an observed height was obtained by stem analysis. Both the taper in the diameter inside the bark and the diameter outside the bark were investigated in this study.

Tapering in the diameter outside the bark

Comparisons among all models indicated that for the overall tree bole, except for model equation (8), a small average bias in diameter was detected in both the fitting dataset (Table 3) and validation dataset (Table 4). However, the difference in the absolute average bias appeared to be greater for all models. In relative terms, the mean biases in percentage were under 4% for all equations expect equation 8 in both datasets (Tables 3, 4). As evaluated by the first 3 criteria considered simultaneously, the trigonometric taper modeling approach was the worst in describing the taper of the entire tree bole, followed by the sigmoid form approach. On the other hand, segmented polynomials, a variable-form stem profile and a polynomial form with a higher-order approach were preferred to estimate the taper in the entire stem (Tables 3, 4). As a biased estimator with small variance may be preferable to an unbiased estimator with a large variance, the MSE was also used to evaluate

Table 3. Comparison of bias and standard error of estimating the diameter outside the bark among models for the entire tree bole based on the fitting data

Model equation	Bias			Standard error (cm)	MSE (cm^2)	Rank
	Average (cm)	Percentage average (%)	Absolute average (cm)			
1	0.15	2.03	1.30	1.86	3.46	5
2	0.02	0.56	0.91	1.90	3.65	6
3	0.16	3.21	1.64	2.40	5.76	7
4	-0.13	-3.40	0.98	1.41	1.98	2
5	-0.26	-3.81	1.26	1.77	3.11	4
6	-0.17	-2.40	1.06	1.50	2.24	3
7	-0.05	-0.42	0.87	1.28	1.62	1
8	-2.02	-27.80	3.11	4.20	17.59	8

MSE, mean square error.

Table 4. Comparison of bias and standard error of estimating the diameter outside the bark among models for the entire tree bole based on the validation data

Model equation	Bias			Standard error (cm)	MSE (cm^2)	Rank
	Average (cm)	Percentage average (%)	Absolute average (cm)			
1	-0.18	0.69	1.52	2.43	5.92	5
2	0.17	0.27	1.51	2.55	6.53	6
3	-0.16	2.82	1.99	2.72	7.37	7
4	0.35	-2.23	1.65	2.38	5.85	4
5	-0.85	-3.87	1.57	2.23	4.94	2
6	-0.53	-3.56	1.36	1.97	3.82	1
7	-0.69	-0.70	1.34	2.36	5.46	3
8	-2.72	-29.20	3.70	5.06	25.5	8

MSE, mean square error.

the diameter estimators (Coble and Wiant Jr 2000). The rank of models based on MSE is listed in Tables 3 and 4.

While there was no overall consistency found in the rank for both data sets, the relative advantage among models was quite similar in the 2 cases. Overall, the performance of the validation dataset was slightly inferior to that of the fitting dataset. All models used, except for equation (8), displayed no bias with respect to the total height. However, only equation (2) displayed no bias with respect to DBH.

Table 5 summarizes the average bias for all models for different segments of the stem along the tree bole. In the entire-bole system approach, only 1 mathematical formula was used to represent the entire tree file, therefore, it is very difficult to fit it effectively. Even the most complicated models may fit 1 portion of the tree very well but show considerable bias elsewhere (Demaerschalk and Kozak 1977). This study shows that neither model with the sigmoid-form approach could predict root swell well (bottom end of the stem) even though they performed well for other parts of the stem bole (e.g., the middle part or the canopy end of the stem axis) (Table 5). It is therefore suggested that using different functions for the lower and upper bole can considerably improve the prediction system.

Compared with other types of taper models, while the polynomial with low power is the easiest model to be fitted, its goodness of fit or predicting ability is quite poor, especially in describing the lower part of the stem such as the stump or sections at 0.3~1.3 m, 1.3 m~0.1 HT, and 0.1~0.4 HT. However, the handicap in describing the lower part of a tree bole can be highly improved by using high powers of relative height (Table 5). In section 0.3~1.3 m, for example, the average bias (2.26) for equation (4) was only about 1/2 of that revealed by equation (3) (5.25). The improvement in using a high power was more noticeable if the average bias in percentage criterion was used (Table 6). In equation (4), only the 3/2 and 3rd powers were needed to describe the upper 4/5 of the bole with a similar accuracy as shown in equation (3) (Bruce et al. 1968).

All graphs clearly showed that the overall shape of the trees is very much the same for all size classes (Fig. 2). Therefore, the inflection point (the point where the tree form changes from neiloid to paraboloid) appears to be at a more or less constant relative height, regardless of the size class. Because the diameter at the inflection point is not affected by root swell and is located in the

Table 5. Average biases of estimating the diameter outside the bark (cm) from the ground to the top for all models based on the validation data

Height from the ground	Sample size	Model equation							
		1	2	3	4	5	6	7	8
0.3 ~1.3 m	22	2.29	6.51	5.25	-2.26	-1.84	-1.87	-2.88	7.74
1.3 m ~ 0.1 HT	24	-2.19	-0.15	-1.61	0.60	-1.12	-0.91	-0.62	0.80
0.1 ~ 0.2 HT	45	-1.82	-1.08	-2.54	1.32	-0.19	-0.07	0.20	-1.01
0.2 ~ 0.4 HT	88	-1.01	-1.02	-2.21	0.61	-0.61	-0.76	-0.17	-2.63
0.4 ~ 0.6 HT	89	0.15	0.13	0.35	0.71	-0.27	-0.51	-0.04	-3.49
0.6 ~ 0.8 HT	88	0.60	0.69	1.14	0.56	-0.70	-0.40	0.54	-4.80
0.8 ~ 1.0 HT	105	0.25	0.06	0.86	-0.26	-1.68	-0.29	0.31	-4.15

HT, total height.

Table 6. Average biases of estimating the diameter outside the bark in percentage (%) from the ground to the top for all models based on the validation data

Height from the ground	Sample size	Model equation 1	2	3	4	5	6	7	8
0.3 ~ 1.3 m	22	5.49	15.07	12.32	-4.10	-4.11	-4.17	-5.47	18.03
1.3 m ~ 0.1 HT	24	-5.31	-0.24	-3.90	1.07	-1.88	-1.29	-0.82	2.66
0.1 ~ 0.2 HT	45	-4.95	-3.05	-7.21	2.25	-0.11	0.25	0.72	-2.54
0.2 ~ 0.4 HT	88	-2.70	-3.03	-6.99	0.35	-1.35	-1.92	-0.90	-8.69
0.4 ~ 0.6 HT	89	0.47	0.17	-1.66	-0.06	-1.52	-2.64	-2.50	-16.28
0.6 ~ 0.8 HT	88	1.88	2.25	5.99	-1.80	-7.5	-4.97	-1.22	-36.21
0.8 ~ 1.0 HT	105	5.51	-0.08	16.03	-9.17	-37.8	-6.76	1.87	-80.31

HT, total height.

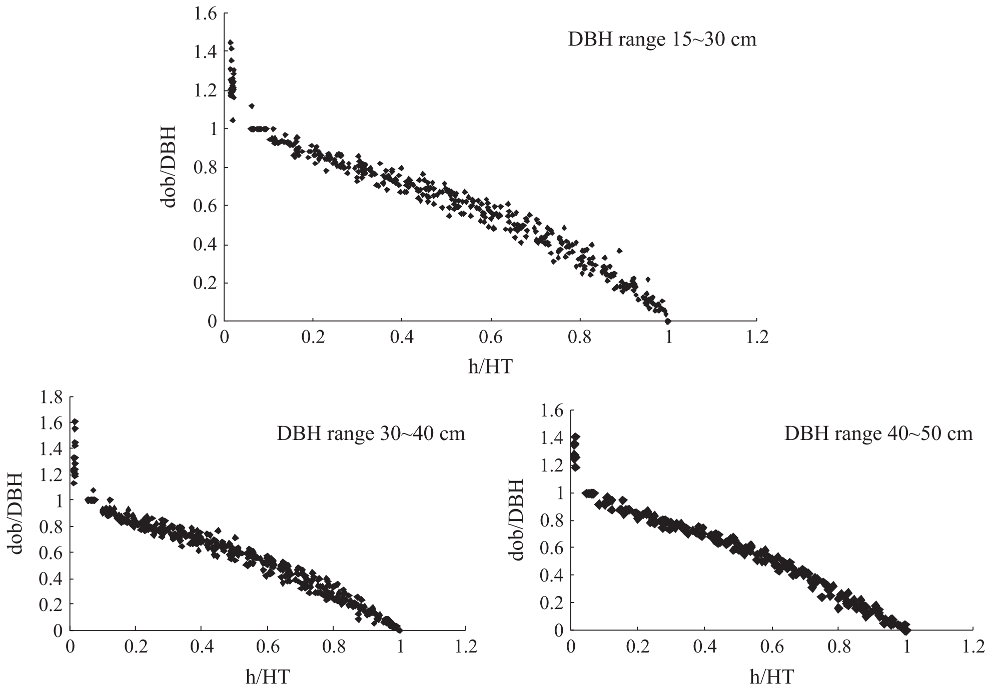

Fig. 2. Plotting of tree profile for different size classes in sample trees. Dob is the diameter outside the bark at a given height (h), DBH is the diameter outside the bark at the breast height, and HT is the total tree height.

least changing part of the stem, it is the base diameter for the taper system (Demaerschalk and Kozak 1977). The profiles above and below the inflection point are so smooth that 2 models were considered to be sufficient to adequately describe the tree profile. These 2 functions can be linked at the inflection point and be conditioned to be continuous at that point.

In segmented polynomials with sub-model taper models, a considerable number of different joints were fitted to the data and the 1 with the least MSE was picked up in both equations (5) and (6). This study shows a slight improvement with the 2-joint submodel over the 1-joint submodel in prediction ability over most segments, but with a notable promotion on the segment close to the tip. Therefore, it was used to compare with models of other approaches. This is because formulating a taper model by splining together 3 polynomials could describe the profile of 1 of the 3 segments for each polynomial (Valentine and Gregoire 2001). The joints in equation (6) shown in this study are quite similar to those in the plantation taper equation study (Max and Burkhart 1976).

Kozak (1988) mentioned that a general shape of diameter tapering from ground to top can be described using a single continuous function as the base with an exponent that changes along the stem to account for the neiloid, paraboloid, and conic forms. Such a power function eliminates the necessity of developing segmented taper functions for different portions of the stem in order to reduce local bias (Bi 2000). A study of the various forms of the exponent indicated that the exponent can be expressed as a multiple curvilinear regression (Kozak 1988). The exponent from ground to top for different sizes of trees from Taiwania plantations in this study indicated that the exponent value is variable at different relative heights from the ground (Fig. 3). The general trend of the exponent was similar for different tree sizes of Taiwania. It can be observed that the inflection point seems to be quite constant (almost 0.25) regardless of the tree size, and the curves with higher DBH/HT values lie above those with lower DBH/HT values.

In the variable-form taper model, an inflection point at 25% of the total height was chosen. Setting the location of inflection at 15, 20, 30, and 35% of the total height had

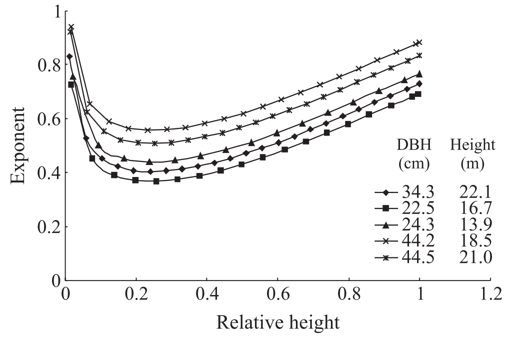

Fig. 3. Changes in exponents for 5 Taiwania plantation trees. DBH, diameter at the breast height.

little effect on the predictive property of the model. This trial is consistent with what Perez et al. did (Perez et al. 1990). While Kozak's (1988) taper function is quite precise, Kozak (1998) mentioned that there is still room to improve its effectiveness by incorporating an additional upper stem diameter measurement. However, caution must be taken to reduce errors in both stem diameter and height from the ground measurements.

By explicitly taking the various exponents into account in depicting the taper profile, the variable-form taper equations fit the data significantly better, especially for the lower part of the stem, than the entire-bole estimating system such as the sigmoid form (Table 5).

The root swell (the section of bole at a height between 0.3 and 1.3 m) is the part of the stem that is most difficult to predict with tapering modeling. Most models tend to underestimate the region below DBH with a substantial divergence in amount (Table 5), and the mean bias percentage ranged from an 18.03% underestimate to an 5.47% overestimate (Table 6). The accuracy in the predicting the tree root swell can be significantly improved through use of a polynomial with a much higher power, segmented polynomials, or the variable-form taper models.

The advantages of segmented polynomials or variable-form taper models are also shown by the evaluation criteria of both the average absolute biases and standard errors of the estimates (Tables 7, 8). It is quite interesting to determine the relative height of the crown base height because this is a point where a great difference occurs in taper rates along the bole. This study showed that the average relative height of the crown base of Taiwania is about 40% of the total height. Because the rapid taper occurs, it appears that the precision of estimates at the lowest portion of the bole is the worst, followed by segments within the live crown (i.e., the segment from the tip to a point at 40% of the total height) for most models. One can obtain a higher precision in predicting the segments between the root swell and the base of the crown.

While the variable-form taper models and segmented polynomial models are very powerful at predicting the diameter at a given height, they both have 2 primary weaknesses: (1) numerical integration methods must be used to calculate the volume; and (2) iterative methods must be used to find the merchantable height to a given diameter. But, both shortcomings are relatively easy to solve with modern computing equipment or facilities (Perez et al. 1990).

Table 7. Average absolute biases of estimating the diameter outside the bark (cm) from the ground to the top for all models based on the validation data

Height from the ground	Sample size	Model equation							
		1	2	3	4	5	6	7	8
0.3 ~ 1.3 m	22	2.63	6.51	5.25	3.2	2.70	2.72	3.72	7.74
1.3 m ~ 0.1 HT	24	2.19	0.15	1.61	0.60	1.43	1.31	0.86	0.80
0.1 ~ 0.2 HT	45	1.87	1.12	2.56	1.52	0.77	0.78	0.72	1.35
0.2 ~ 0.4 HT	88	1.47	1.35	2.33	1.58	1.23	1.30	0.96	2.71
0.4 ~ 0.6 HT	89	1.52	1.51	1.67	2.09	1.55	1.63	1.73	3.67
0.6 ~ 0.8 HT	88	1.80	1.85	2.08	2.09	1.80	1.69	1.82	4.92
0.8 ~ 1.0 HT	105	0.81	0.79	1.08	0.95	1.83	0.91	0.82	4.36

HT, total height.

Table 8. Standard error of the estimating the diameter outside the bark (cm) from the ground to the top for all models based on the validation data

Height from the ground	Sample size	Model equation							
		1	2	3	4	5	6	7	8
0.3 ~ 1.3 m	22	3.73	7.97	6.69	4.23	3.67	3.94	6.21	7.84
1.3 m ~ 0.1 HT	24	3.03	0.63	2.31	2.08	2.64	2.63	1.88	1.94
0.1 ~ 0.2 HT	45	2.39	1.48	3.12	4.07	1.05	1.06	1.10	1.87
0.2 ~ 0.4 HT	88	1.92	1.73	3.02	3.46	1.60	1.72	1.34	3.39
0.4 ~ 0.6 HT	89	2.10	2.12	2.22	4.25	2.12	2.21	2.98	4.61
0.6 ~ 0.8 HT	88	2.50	2.52	0.67	3.70	2.70	2.58	3.28	6.27
0.8 ~ 1.0 HT	105	1.24	1.21	1.49	1.47	2.61	1.39	1.50	6.09

HT, total height.

Generally, tree taper variations might be caused by differences in stand, tree, and site characteristics as well as stand history (Larson 1963, Muhairwe et al. 1994). However, Muhairwe et al. (1994) showed that incorporating stand, tree, and site variables into the variable-form exponent of Kozak's (1988) taper equation did not result in large improvements in predicting the diameter. Therefore, the cost of measuring these additional variables is not justified. No attempt was made to evaluate the effects of stand density or site characteristics on tree taper in this study because the proper data are not available.

Quite a few taper studies are available in Taiwan. Compared with other studies, the ability to predict the bole profile of Taiwania plantation trees demonstrated in this study is not as good as those carried out by other studies, particularly with respect to segment estimation. This is because the sample size of measurements available in this study was quite smaller than those in other studies (Biging 1984, Perez et al.1990).

In conclusion, although there is still a slight bias near ground level (< 6%), for the 3-segmented polynomial models and variable-exponent models, their prediction of diameters outside bark had < 3% bias over most of the lengths of the trees. Therefore, these 2 tree profile prediction systems describe the tree shape more realistically than any other taper system we tested.

Tapering in the diameter inside the bark

The diameter inside the bark (DIB) at an observed height was obtained by using the linear regression of the DIB on the diameter outside the bark (DOB) based on the data from the stem analysis. The resultant regression was DIB = -0.274 + 0.979*DOB with r^2 = 0.996. In general, comparisons of model performance discussed with tapering in DOB sections are valid for DIB as well. The estimated coefficients and associated standard errors for preferred models are listed in Table 9 for practical reference purposes.

The total volume or merchantable volume can be obtained by mathematically integrating the taper equation from ground to tip or to a merchantable height. A compatible relationship among them accordingly exists (Reed and Green 1984, Jordan et al. 2005). Since the purpose of this study was exclusively focused on the taper, no attempt to consider the mathematical relationship among taper, total volume, and merchantable volume was made. However, because of its great

Table 9. Estimated coefficients and their standard errors (in parentheses) of the estimated diameter inside the bark for models based on the fitting data

Model equation	Parameters								
	b_1	b_2	b_3	b_4	b_5	b_6	b_7	b_8	α_1	α_2
4	9.39506	-0.73072	17.25352	-16.13514	15.47089	-139.06228				
	(0.0841)	(0.249795)	(4.96539)	(2.09701)	(2.95451)	(35.91704)				
6	-3.03218	1.42503	-1.01493	87.47377					0.1	0.8
	(0.5542)	(0.3221)	(0.37346)	(2.48649)						
7	0.7723	1.0315	0.9973	1.6217	-0.3671	2.2974	-1.2737	0.1872		
	(0.1845)	(0.0965)	(0.00295)	(0.1908)	(0.0443)	(0.4477)	(0.2378)	(0.0148)		

Model 4, high-order polynomial approach.
Model 6, 3-segmented polynomial approach.
Model 7, variable-form approach.

importance, development of a system of a compatible volume-taper model for Taiwania plantation should be pursued in the future.

CONCLUSIONS

As the demand for forest products is generally increasing with time while the land base for producing forest products is decreasing around the world, accurate estimates of individual tree volumes are needed to make a better determination the potential value of standing trees for use as solid wood products, fiber, or fuel. The taper system is used mainly to predict volumes inside and/or outside the bark of logs, tops, and stumps on standing trees. Taper function provides the way to predict the diameter at any height and the height at any diameter based only on tree measurements of DBH and the total height commonly taken in inventories, thereby enhancing the value of any forest inventory from the prediction of total to merchantable volumes to any merchantability limits.

The root swell (0.3 to 1.3 m) is a part of the stem that is most difficult to predict in tapering modeling. However, the precision and accuracy in predicting the tree root swell can be significantly improved through the use of segmented polynomials or variable-form taper models. Together with their excellence in predicting the stem diameters over most of the segments of the trees, therefore, the 3-segmented polynomials or the variable-form taper models are thought to be superior to others in estimating the bole profile of Taiwania plantation trees.

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